Directional Cheap Talk in Electoral Campaigns

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Abstract

Can campaign communications credibly transmit information about candidates' policy intentions? To answer this question, I develop and analyze a game-theoretic model of campaign communication in a two-candidate majority rule election with multidimensional policies. Candidate and voter preferences are private information and campaigns consist of both candidates sending cheap talk messages in order to communicate information about their preferences. The game possesses equilibria involving informative campaign messages which reveal information about the directions of the candidates' ideal points from the center of the policy space but leave the voters uncertain about which candidate is more extreme.

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Formal political theory has taken a dismal view of communication by politicians: If talk is cheap in elections then candidates are willing to say whatever it takes to get elected. Therefore, the argument goes, nobody should expect campaign speeches to convey useful information. This argument does not comport with empirical evidence, which suggests that what politicians say is a good predictor of what they will do in office.[1]

The tension between *cheap talk* models of electoral communication and empirical work on electoral communication is widely recognized. Recent books on political communication draw this contrast clearly:

> Perhaps most surprising of all is that constituents trust the content of legislators' messages. After all, in a principal-agent relationship, this sort of reporting would be costless cheap-talk that strategic principals would dismiss as uninformative signaling. (Grimmer, 2013, p. 24-25)

> ...contrary to the conventional wisdom that candidates' appeals are just 'cheap talk,' campaigns actually have a lasting legacy in the content of representatives' and senators' behavior in office. (Sulkin, 2011, Frontmatter)

Since political scientists presume that cheap talk is uninformative in elections, we often take informative statements by candidates as evidence of the effects of some other mechanism such as repeated elections[2] or screening by political parties.[3]

I argue that the conventional wisdom about cheap talk in elections is misleading. Using a game-theoretic model of multidimensional electoral competition with asymmetric information about candidates' and voters' preferences, I show that campaign speeches may be informative even in one-shot elections when talk is cheap. The key insight driving this conclusion is that candidates can reveal *directional* information about their preferences without

[1]For instance, Sulkin (2011) and Grimmer (2013) each show evidence that legislators' communications to constituents are good predictors of their policy priorities once in office.

[2]See, for instance, Harrington (1993) and Aragonès, Postlewaite and Palfrey (2007).

[3]See, for instance, Snyder and Ting (2002) or Ashworth and Bueno de Mesquita (2008).

revealing which candidate is more moderate. For instance, candidates may truthfully make statements such as “I am economically liberal, socially conservative, and a hawk on national defense.” Since the candidates do not know the exact preferences of the electorate they are uncertain *ex ante* about which messages will cause them to win the election. However, once the voters’ preferences are known, the vote goes clearly for one candidate or the other and the information in the messages improves the welfare of the voters.

The results contribute to scholarly understanding of elections in two ways. First, the results imply that scholars should think of informative campaigns as a natural outcome of elections rather than as an aberration induced only by some other mechanism. This point also affects theories about elections that do *not* resemble cheap talk since uninformative signaling has served as the implicit counterfactual against which we have evaluated the effects of institutions presumed to cause informative campaigning.

Second, the model provides qualitative predictions about the content of candidates’ campaign messages. Candidates describe their policy positions in terms of their direction away from the center. On the surface this seems contrary to intuition about spatial competition; after all, all candidates would prefer to be viewed as centrists. Announcements of centrist policy positions, however, are not credible. If voters believed candidates when they claimed to be centrists, then *all* candidates would make such claims and voters would be irrational to believe them. In previous models the same logic meant that informative cheap talk was impossible in elections, but here it simply changes the nature of the information that voters receive: voters can distinguish economic liberals from economic conservatives and social conservatives from civil libertarians, but not moderates from extremists. Furthermore, a special case of the model provides predictions about issue selection in campaigns: candidates may endogenously define issue dimensions along which they can more credibly reveal their positions.

1 Elections with incomplete information

This study contributes to the existing theoretical literature on elections with incomplete information. The Hotelling-Downs spatial model of electoral competition assumed that candidates were required to fulfill their campaign promises. As a result, classic models sidestepped a central issue concerning elections, which is whether candidates have an incentive to reveal truthful information about their policy intentions. To address this issue, scholars developed models of electoral competition between policy-motivated candidates in which voters had incomplete information about candidates' policy intentions. These models explained the heart of candidates' credibility problem very clearly: since candidates have an incentive to say whatever it takes to win, a rational voter cannot always take candidates' messages at face value.

1.1 Cheap talk and uninformative signaling

The foundation of theories of electoral communication is the cheap talk model. In cheap talk models there are no costs to the candidate for lying, no verifiability of information, and no mechanism for making politicians accountable for inaccurate information. Since electoral institutions can be put in place to make lying costly and verifiable or to provide accountability, the cheap talk model is foundational in the sense that it provides a baseline against which the effects of these institutions can be judged. Early work on electoral cheap talk reached a stark conclusion: when talk is cheap, there can be no information transmission from candidates to voters (Harrington, 1992). It follows that any meaningful campaign communication is attributable to institutional deviations from the cheap talk setting.

The study of information in elections is organized in part around the apparent failure of information transmission under cheap talk. The theoretical literature has focused on providing institutional and behavioral mechanisms to permit information transmission from candidates to voters. The most straightforward mechanism is to assume that candidates face costs for lying. Substantively, lying costs must often be fairly high in order to induce informative campaign messages. For instance, Banks's (1990) result establishing partially informative campaign messages requires that lying costs are high enough that some extreme types would rather lose the election than falsely announce a moderate position. However, Callander and Wilkie (2007) and Kartik and McAfee (2007) each show that the existence of some candidates for whom lying is costly can induce informative campaigning from cheap talking candidates.[4]

1.2 Institutions promoting information transmission

The theoretical literature also analyzes the role that various electoral institutions play in encouraging informative campaigning. For instance, repeated elections may encourage truth telling by allowing voters to punish liars and party labels may provide a costly signal that lends credibility to certain policy claims. However, when we interpret these theories against the baseline of uninformative cheap talk, it is difficult to fully account for relevant empirical facts. Furthermore, we risk overestimating the causal effects of these institutions for promoting information transmission when we treat uninformative signals as the counterfactual outcome in the absence of those institutions.

The hypothesis that repeated elections create incentives for truth-telling is intuitively appealing since incumbents frequently face accusations of breaking their campaign promises. Formal theorists have explored this mechanism. Harrington (1993) showed that reelection incentives can induce truthful campaigns provided that voters' preferences are sufficiently responsive to incumbents' performance in office. Aragonès, Postlewaite and Palfrey

[4] The main model in Kartik and McAfee (2007) assumes that candidates can commit to platforms, but in an extension of the model – which corresponds roughly to a special case of Callander and Wilkie's (2007) model – they show that there is a partially informative equilibrium with no commitment.

(2007) similarly showed that reelection incentives can induce politicians to keep campaign promises in an infinite-horizon model of repeated elections. Though reelection incentives are a plausible contributor to incentives for truthful campaigns, empirical evidence appears inconsistent with the hypothesis that politicians do what they promise *solely* because of reelection concerns. For instance, members of Congress are more likely to keep campaign promises when they are *not* electorally vulnerable (Sulkin, 2011).[5] Furthermore, citizens often pay little attention to candidate behavior in office and frequently cannot identify their legislators' votes even on high profile roll calls (Dancey and Sheagley, 2013) so voters are unlikely to respond to politicians' behavior enough for reelection incentives alone to account for the informational value of campaigns. This paper shows that candidates can make credible policy promises without any accountability but that a lack of accountability limits the nature of information candidates will reveal.

Political parties may also provide a mechanism for informing voters. According to theories of informative party labels, political parties adopt practices that make it costly for candidates to adopt party labels if they do not agree with the party's positions (Snyder and Ting, 2002; Ashworth and Bueno de Mesquita, 2008). Critically, these theories depend on the existence of party affiliation costs, which are interpreted as arising from party discipline or from efforts by parties to screen candidates. Thus, weak parties who do not screen candidates or impose party discipline should not be associated with informative labels. Furthermore, candidates should not be expected to reveal information beyond what is communicated by their party labels. In contrast, I show that party screening is not necessary to make candidates adopt informative labels.

[5] Sulkin (2011) alludes to the same point when discussing this finding: "The most common view in the literature on legislative behavior and representation is that more vulnerable legislators...should also be the mostly likely to engage in high levels of reelection-promoting behavior. This argument suggests a negative relationship between vote shares in the previous election and subsequent levels of promise keeping....though, this hypothesis presumes that legislators do not want to follow through on their appeals, and do so only because of electoral imperatives. If they raise issues at least in part because of their genuine interest in them, this logic unravels a bit" (p. 132).

1.3 Theoretical precedents for my argument

A key difference between my analysis of electoral cheap talk and some previous work is that candidates' ideal points are drawn from a continuous rather than a discrete distribution. Harrington (1992), for example, points out that a discrete type space for candidates is necessary for the result that cheap talk is always uninformative in elections. If utility functions and type distributions are both continuous, he suggests that "there always exist two messages which yield the same probability of winning the election" (p.143). Thus, Harrington anticipated the one-dimensional result in this paper (Theorem 1). I more fully develop this intuition before extending it to multiple dimensions in a variety of ways.

Though this study provides a novel theoretical argument for informative cheap talk in elections, other recent work has reached similar conclusions using other mechanisms. Van Weelden and Kartik (2015) show that candidates may reveal their preferences to voters using cheap talk as a way to credibly commit to future policies and avoid temptations for pandering. The basis for informative communication in Van Weelden and Kartik (2015) is different from this paper: in their study, candidates reveal themselves to be non-congruent in order to eliminate the incentive to choose bad policies later on. In my model, candidates plan to implement their ideal points no matter what they said in the election.

The mechanism for information transmission in my model is most closely related to a non-electoral model by Chakraborty and Harbaugh (2010). One way of understanding why *electoral* cheap talk is especially difficult is that candidates' preferences over voters' actions are independent of their type. Though persuasion often requires that the speaker's preferred actions respond to information in a similar way to the receiver's (Crawford and Sobel, 1982), candidates prefer to be elected no matter their policy intentions. However, Chakraborty and Harbaugh's (2010) model of multidimensional cheap talk shows that senders with state-independent preferences can informatively make comparative statements across different dimensions. The results in this paper (especially Theorem 2) illustrate a similar logic.

2 The model

I consider a model of an election in which candidates are unable to commit to policy platforms. Candidates send campaign messages to convey information to voters about their policy intentions. The policy space is $\mathbb{R}^d$, with $d \geq 1$. The set of players consists of two candidates, denoted A and B, and a finite set N with an odd number n of voters. The ideal policy of each player $i \in \{A\} \cup \{B\} \cup N$ is denoted $\mathbf{z}_i \in \mathbb{R}^d$. Each player's preferences are represented by a quadratic Euclidean utility function $u(\mathbf{x}, \mathbf{z}_i) = -||\mathbf{x} - \mathbf{z}_i||^2$ where $\mathbf{x}$ is the policy that is implemented and $||\cdot||$ is the Euclidean norm.

The election proceeds as follows. First, both candidates send costless and public campaign messages $\mathbf{m}_A, \mathbf{m}_B \in \mathbb{R}^d$ to the voters about their policy preferences. The results do not depend on whether the candidates' messages are simultaneous or sequential so I leave this aspect of the model open to interpretation. Second, the voters observe the messages, update their beliefs about the ideal points of the candidates, and each voter votes in favor of one of the candidates. Finally, the candidate with the majority of votes wins the election and implements her ideal policy.[6]

All players' ideal points are private information. The players believe that the candidates' ideal points are independently and identically distributed from an absolutely continuous probability measure F with density f and that voters' ideal points are independently and identically distributed according to an absolutely continuous probability measure G.

A strong assumption in this model is that the candidates' ideal points have the *same*

[6]The fact that candidates are policy-motivated and cannot commit to policy platforms makes this model similar to citizen-candidate models of electoral competition (Osborne and Slivinski, 1996; Besley and Coate, 1997).

prior distribution. I maintain this assumption for two reasons. First, the case of *ex ante* identical candidates best facilitates comparison to previous work (e.g. Harrington (1992), Banks (1990)). Second, some initial differences between candidates are driven by candidate choices (for example, party labels) which much of the literature treats as products of equilibrium rather than exogenous characteristics. To focus on informational incentives for candidates to differentiate themselves, it is useful to start with undifferentiated candidates. However, the model does not capture differences in the appearance or background of candidates that may cause beliefs about the candidates to diverge.

I consider symmetric perfect Bayesian equilibria in weakly undominated strategies. A (pure) strategy for the candidates is a function $\sigma : \mathbb{R}^d \to \mathbb{R}^d$ mapping each possible ideal point into a campaign message. Thus, $\sigma(\mathbf{z}) = \mathbf{m}$ means that a candidate with an ideal point of $\mathbf{z}$ sends the message $\mathbf{m}$. The focus on symmetric equilibria means that candidates A and B would send identical messages if they had the same ideal point. A strategy for each voter is a function $v : (\mathbb{R}^d)^3 \to [0,1]$ mapping the voter's ideal point and both candidates' messages into a probability of voting for candidate A. For example, if $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1$, then a voter with ideal point $\mathbf{z}_i$ votes for candidate A with probability 1 when the candidates send the message $\mathbf{m}_A$ and $\mathbf{m}_B$. The focus on weakly undominated strategies means that voters always choose the candidate that, if elected, would give them the highest expected utility. This rules out the possibility that voters choose their least-favored candidate when they are not pivotal. Finally, let $\mu(\mathbf{m})$ denote the beliefs of the voters about the ideal point of a candidate who sends the message $\mathbf{m}$. Voters' beliefs about a candidate depend only on that candidate's message.

An equilibrium to the game is a profile of strategies and beliefs such that:

1. $\sigma(\mathbf{z})$ maximizes each candidate's expected utility given that the voters' play v and the other candidate plays σ,

2. $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1$ if A gives a voter with ideal point $\mathbf{z}_i$ a higher expected utility under beliefs μ, $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 0$ if B gives that voter a higher expected utility under beliefs μ, and $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1/2$ if the expected utilities for a voter with ideal point $\mathbf{z}_i$ are equal for both candidates under beliefs μ, and

3. μ is consistent with Bayes' rule along the equilibrium path: letting $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \sigma(\mathbf{z}) = \mathbf{m}\}$, we have

$$\mu(\mathbf{m}) = \begin{cases} f(\mathbf{x})/[\int_{S(\mathbf{m})} f(\mathbf{x})] & \text{if } \mathbf{m} \in S(\mathbf{m}) \\ 0 & \text{otherwise.} \end{cases}$$

whenever $S(\mathbf{m})$ is non-empty.[7]

The meanings of the candidates' messages are determined only by the sets of types who would send those messages in equilibrium. Therefore, the analysis is concerned with the the sets $S(\mathbf{m})$ induced by the messages rather than the messages themselves. If $S(\mathbf{m}) = S$, the message $\mathbf{m}$ is meant to convey the information "My ideal point is somewhere in the set S." In reality, the candidate would likely use more natural language such as "I am economically liberal and socially conservative" or "I prefer to increase spending on food stamps twice as much as on science funding." The effect of these messages is to inform the voter by reducing the set of preferences that the candidate might hold.

Since messages are costless, candidates are exactly indifferent between messages in equilibrium. This feature of cheap talk equilibria means that the predictions can be taken as lower bounds on electoral information transmission: the equilibria should be interpreted as statements about how much information transmission is possible when nothing is nudging

[7]As is common in the analysis cheap talk games, I largely ignore off-path beliefs. All of the equilibria could be supported, for instance, by off-path beliefs in which unused messages are interpreted as being identical to the nearest equilibrium message. Alternatively, the candidates' strategies could easily be converted to mixed strategies that eliminate the possibility of off-path signals without changing the equilibrium outcomes.

them toward sending truthful messages. However, the equilibria can be strengthened by departures from the cheap talk assumptions. For instance, if candidates experience any cost of lying (that is, sending a message $\mathbf{m}$ such that $\mathbf{z}_j \notin S(\mathbf{m})$) then the predictions are unchanged except that they become strict equilibria.

I will focus on equilibria in which all pairs of messages give both candidates the same probability of victory. Since the candidates are policy-motivated and care about the policy selected by their opponent when they lose, there are likely to be some perfect Bayesian equilibria that do not have this property. In some strategy profiles, candidates may be willing to accept a lower overall probability of victory in order to reduce the probability of losing to opponent types that are far from their ideal points. As Lemma 1 in Appendix B demonstrates, any profile that equalizes victory probabilities across all pairs of messages is a perfect Bayesian equilibrium to the game. The reason is that every message a candidate could send gives her an equal probability of winning *against all types of opponents*. Therefore, in these strategy profiles, switching to a new message cannot change a candidate's probability of victory *or* her distribution of opponents conditional on losing.

Equilibria that equalize candidates' victory probabilities across pairs of messages are more attractive in a couple of ways. First, these equilibria are robust to adding in office motivation for candidates. Second, equilibria that equalize victory probabilities for all pairs of messages are not sensitive to the sequence of the game: candidates do not want to switch messages after they learn their opponents' message, so the predictions are invariant to the arbitrary choice of whether communication is simultaneous or sequential. In contrast to both of these points, equilibria in which candidates do not maximize victory probabilities can be destroyed by adding office motivation or changing the sequence of the game.

3 Analysis

The analysis uses the model to make two main arguments. First, informative cheap talk equilibria exist. In these equilibria, candidates reveal directional information about their policy intentions but leave voters uncertain about which candidate is more moderate. Second, under common distributional assumptions, the candidates reveal binary positions along orthogonal ideological dimensions. Under stronger assumptions, candidates perfectly reveal the directions of their ideal points from the center.

3.1 Existence of informative equilibria

In order to argue for the existence of informative cheap talk in elections, I will begin by characterizing binary announcement equilibria in which candidates reveal themselves to be in one of two camps based on the realizations of their ideal points. The following examples illustrate such equilibria in one and two dimensions.

Example 1. Consider a one-dimensional policy space – for example, a left-right fiscal policy scale – and assume that candidates' and voters' ideal points are drawn from normal distributions centered at zero.[8] In this setting, there is an equilibrium in which each candidate accurately reveals whether her ideal point lies to the left or the right of zero. Thus, the candidates communicate directional information about their preferences without revealing the intensity of their preferences in that direction.

To see why this is an equilibrium, consider what happens when one candidate announces "My ideal point is left of center" and the other announces "My ideal point is right of center" as displayed in Figure 1. Given these messages, the voters still expect the candidates to be equally extreme. As a result, left-of-center voters prefer the left-wing candidate

[8]The distributional assumptions are simply to make the example more concrete. As we will see, they are not required for constructing the equilibrium.

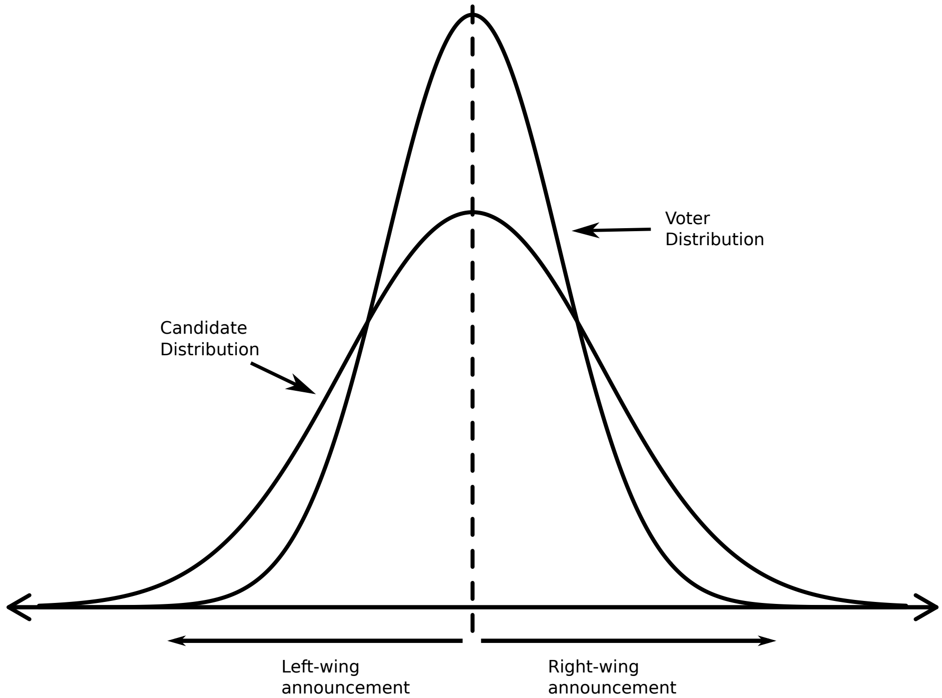

Figure 1: A one-dimensional directional equilibrium from Example 1.

and right-of-center voters prefer the right-wing candidate. Since the distribution of voters' ideal points is symmetric and the candidates remain uncertain about voter preferences, this means that each candidate expects to win the election with a probability of one half. Similarly, if both candidates are on the same side then they will make the same announcement, in which case all voters are indifferent between candidates. Thus, the voters may randomize between candidates and each still expect to win with probability one half. In either case, neither candidate has an incentive to deviate to a dishonest campaign message since the probability of winning is constant for all possible pairs of messages. Furthermore, even though the candidates have the same expected probability of winning before they know the voters' ideal points, the voters gain from campaign speech in expectation since the ex post winner of the election is the candidate that would move policy in the direction preferred by a majority of voters. ■

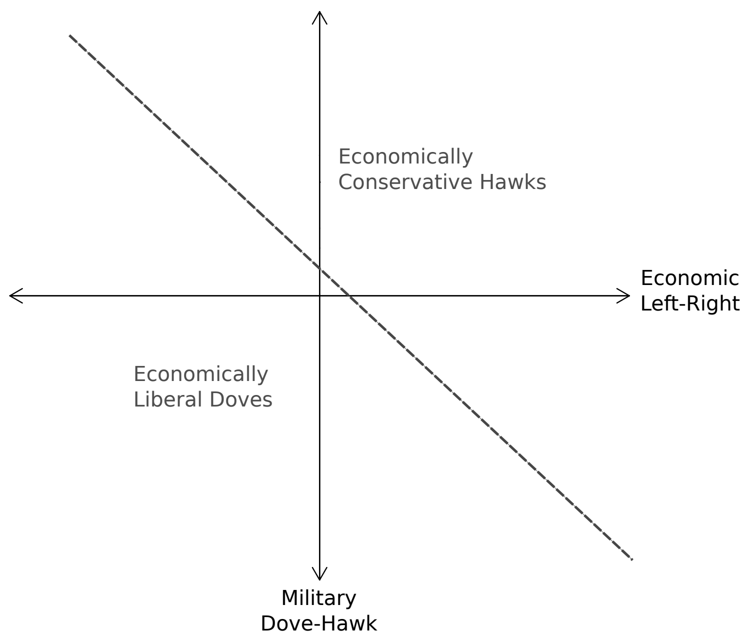

Figure 2: A binary announcement equilibrium from Example 2.

Example 2. Now assume that the policy space is two dimensional. In addition to fiscal policy, voters care about candidates' stances on foreign policy, which can be summarized by a scale along which higher numbers are more interventionist. Candidates' and voters' ideal points are distributed according to some continuous distributions on $\mathbb{R}^2$. In this situation, there is an equilibrium of the following form: a line divides the policy space into two half spaces and both candidates accurately reveal which half space contains their ideal points. The substantive interpretation of the messages depends on the angle and position of the dividing line which are functions of the ideal point distributions, but one example is displayed in Figure 2. In this example, the line has a negative slope and cuts through the middle of the upper left and lower right quadrants of the space, so we can interpret the messages as distinguishing economically conservative hawks from economically liberal doves.

Following the logic of Example 1, an equilibrium dividing line must equalize the probability of winning for both candidates given any pair of messages. To see that such a line always exist, consider any line ℓ passing through a point $\mathbf{x}'$. For such a line, there is some probability that a candidate revealed to be "above" the line beats a candidate revealed to be "below" the line. If this probability equals one half then it is an equilibrium dividing line. Otherwise, suppose this probability is equal to $P < \frac{1}{2}$. Consider what happens when ℓ is rotated around the axis x'. The relevant probability changes continuously with the angle of rotation. Furthermore, once we have rotated the line 180 degrees, the two sides of the line are simply flipped and the probability that the candidate "above" the line wins over the candidate "below" is equal to $1 - P > \frac{1}{2}$. By continuity, there must have been an angle of rotation that equalized the probability of winning the election for candidates on either side of the line. Furthermore, when the candidates are on the same side, the voters are indifferent and the candidates have the same probability of winning. Therefore, there is an informative equilibrium to this game since the probability of winning is equal for all pairs of messages. ■

The types of equilibria illustrated by Examples 1 and 2 exist under very general conditions. To set up the results, let $P(S,S')$ be the victory probability of a candidate who reveals her ideal point to be in the set S when her opponent's ideal point is revealed to be in the set S'.[9] Theorem 1 states that, when the policy space is one-dimensional, there exists a binary announcement informative equilibrium as long as candidates are worse off when they are revealed to have ideal points on the extreme left or extreme right.[10] This condition is met, for example, when the set of possible candidate ideal points is unbounded or when the bounds are sufficiently far from the mean of the voters' distribution.

Theorem 1. *Let $d = 1$. Suppose there there exists a number $\tilde{z} > 0$ such that $P(\{z : z > \tilde{z}\}, \{z : z < \tilde{z}\}) < \frac{1}{2}$ and $P(\{z : z < -\tilde{z}\}, \{z : z > -\tilde{z}\}) < \frac{1}{2}$. There exists an informative equilibrium in which both candidates reveal whether their ideal points lie to the left or the*

[9]A complete description of $P(\cdot,\cdot)$ is provided in Appendix A.

[10]This is also a *necessary* condition for existence of a cutpoint equilibrium. For instance, if there does not exist a number $\tilde{z} > 0$ such that $P(\{z : z > \tilde{z}\}, \{z : z < \tilde{z}\}) < \frac{1}{2}$, then the candidates would strictly prefer to claim that they were to the right of every possible cutpoint.

right of some cutpoint.

The proof of Theorem 1 is a simple application of the intermediate value theorem: at some point near the extreme left, it must be that voters are more likely to prefer a lottery over candidates to the right of that point than a lottery over candidates to the left. At some point near the extreme right, the opposite is true. Therefore, by continuity, there must be some point in the middle that gives the candidates equal probabilities of victory. This profile equalizes the probability of victory for all pairs of messages.

Existence of informative equilibria extends easily to the multidimensional case. Theorem 2 generalizes the equilibrium from Example 2 to any multidimensional policy space. In more than two dimensions the dividing line from Example 2 is replaced with a hyperplane but the argument is the same: the hyperplane divides the policy space into two half spaces that are equally likely to be preferred by a majority of voters.

Theorem 2. *Let $d > 1$. There exists an informative equilibrium in which both candidates reveal whether their ideal points lie on one side or the other of some hyperplane.*

The equilibrium and proof for Theorem 2 is very closely related to Chakraborty and Harbaugh's (2010) result on comparative cheap talk in two or more dimensions. Like their result, my proof is based on the observation that any hyperplane can be rotated such that it divides the space into two half-spaces providing equal utility to the sender(s). The model in Chakraborty and Harbaugh (2010) includes only one sender so Theorem 2 can be seen as an application of their result to a setting with multiple senders. The border conditions needed for Theorem 1 are not necessary in the multidimensional case since the dividing hyperplane is found by rotating around an axis rather than moving from one extreme of the distribution to another.

The dividing hyperplanes in the construction of Theorem 2 need not be unique. Depending on the distribution of candidates' ideal points there may be many different binary announcement equilibria or even an infinite number (such as under the assumptions in section 3.2.2).

3.2 Equilibrium characterization

Equilibria with binary announcements were used to demonstrate that informative equilibria generally exist. However, in multidimensional policy spaces these may not be the most informative equilibria to the game, as the following examples demonstrate.

Example 3. Consider an election in which voters and candidates have positions on fiscal policy and foreign policy. Suppose players' preferences are correlated: people who support conservative fiscal policy also tend to support interventionist foreign policy. To represent this idea, let ideal points for voters and candidates be drawn independently from a bivariate normal distribution with a mean of $(0,0)$ and a covariance matrix of

$$\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}.$$

The contours of this distribution are represented by the ellipses in Figure 3. In this game, there is an equilibrium with four distinct messages corresponding roughly to "economically left, military moderate", "economically moderate dove," "economically moderate hawk," "economically right, military moderate," as illustrated in Figure 3.[11]

The regions corresponding to each message are defined by rotating the axes until the corresponding dimensions are uncorrelated with each other. Such a rotation is possible due to symmetry properties of the multivariate normal distribution, as I will clarify below. In this new rotated space, each quadrant has an equal expected distance from the mean of the

[11]These message labels are chosen to represent the average candidate in each quadrant. It is possible for an "economically left, military moderate" to be more economically moderate in reality than a candidate adopting the "economically moderate hawk" label.

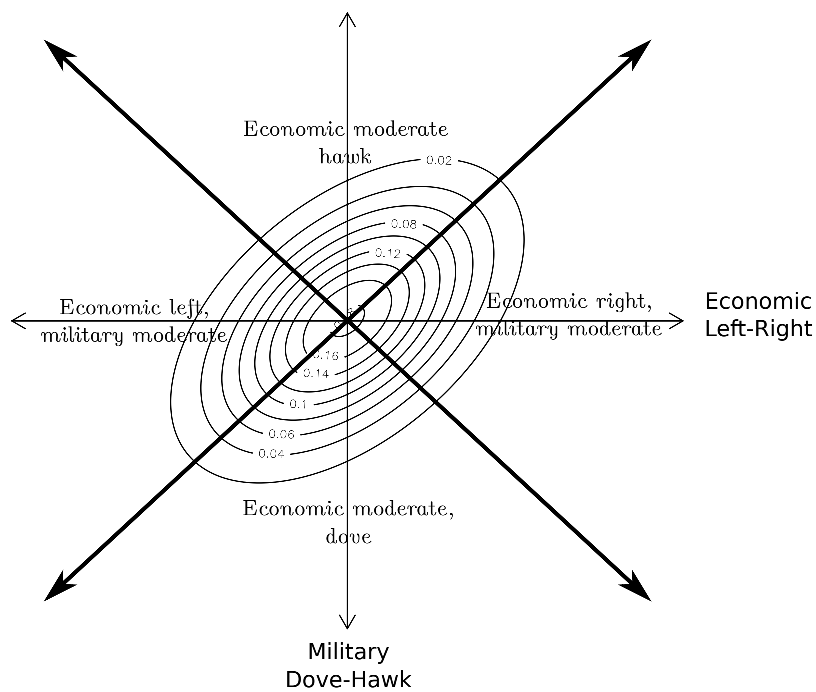

Figure 3: The partially revealing directional equilibrium in Example 3.

voters' ideal point distribution. Therefore, each voter prefers the candidate revealed to be in the nearest quadrant. Following any pair of messages the set of ideal point realizations leading a voter to prefer candidate A to candidate B is divided by a line passing through the origin. Furthermore, the probability that a voter falls to one side of the line is equal to one half. Thus, for any pair of quadrants that the candidates reveal, both candidates expect to win the election with a probability of one half. ■

Example 4. Consider the same election but assume that players' ideal points are not correlated across dimensions. To represent this idea, assume that all players' ideal points are distributed according to a multivariate normal distribution with a mean of $(0,0)'$ and a covariance matrix equal to the 2×2 identity matrix. That is, each player's ideal point in two dimensions consists of two numbers, each drawn independently from a standard normal distribution. The contours of this distribution are represented by the circles in Figure 4. The *direction* of a candidate's ideal point from the center in two dimensions is a ray starting from $(0,0)$ and passing through the candidate's ideal point. This is represented by an angle, ranging from 0 to 2π when measured in radians and ranging from 0 to 360 when measured in degrees. There is an equilibrium to this game in which each candidate perfectly reveals this direction.

To see why this is an equilibrium, consider a situation in which the candidates reveal the two rays shown in Figure 4. Given this information, the voters expect the candidates to be equally extreme in their chosen directions and therefore choose the candidate that would move policy in the direction most similar to their own.[12] Thus, the voters for each candidate are divided by a line passing through the center which, because of the symmetry of the voter distribution, divides the voter distribution exactly in half. Hence, each candidate expects to win the election with a probability of one half. Furthermore, this holds for any two rays

[12]Specifically, they will choose the candidate for whom the angle enclosed by their ideal point and the ray revealed by the candidate has the largest cosine, as is shown in the proof of Theorem 4.

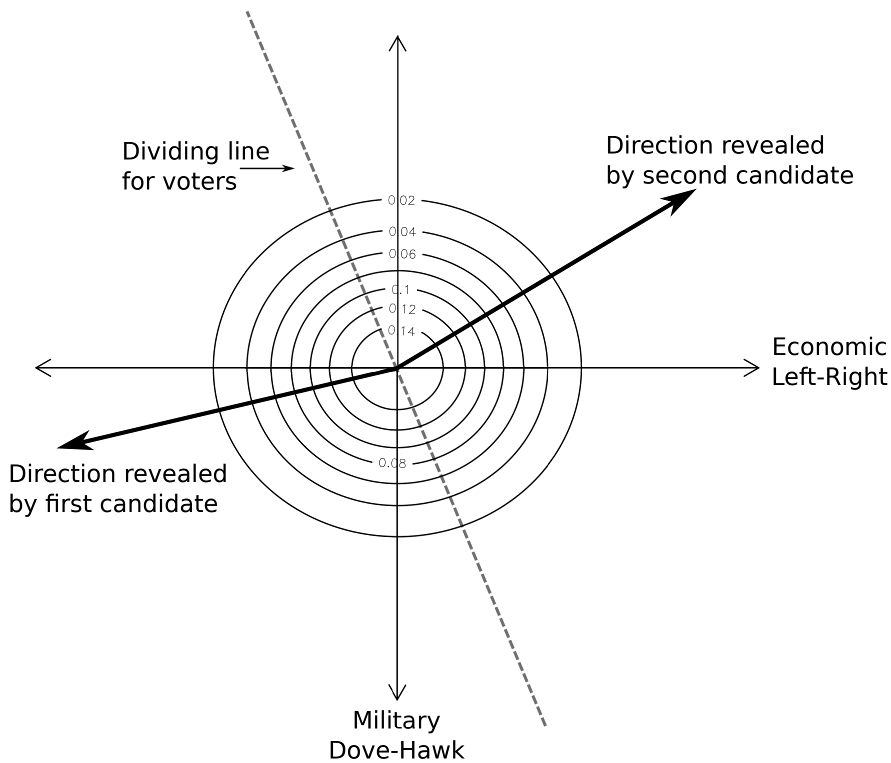

Figure 4: The directional equilibrium in Example 4.

that the candidates may reveal, so it is an equilibrium for all candidates to fully reveal their directions from the center. ■

As Examples 3 and 4 illustrate, electoral cheap talk may reveal very detailed directional information about candidates' preferences under some conditions. These outcomes depend on the distribution of voters' and candidates' ideal points but the required assumptions are consistent with many empirical spatial models of candidate and voter preferences.

3.2.1 Principal orthant equilibria

I first characterize equilibria of the type described in Example 3. In more than two dimensions, the four quadrants of Example 3 are replaced by 2^d orthants but the argument is similar. The assumption required for candidate ideal points is that their probability distribution satisfies a version of symmetry that encompasses, for example, all multivariate normal distributions. Specifically, f must belong to the class of elliptical distributions:

Assumption 1. The density of candidate ideal points f is *elliptically symmetric* around the point $(0,\ldots,0)$ (Fang, Kotz and Ng, 1990): $f(\mathbf{z}) = c \cdot f_0(\mathbf{z}'\Sigma^{-1}\mathbf{z})$ where f_0 is continuous and non-negative and the scale parameter Σ is a real, symmetric, positive definite $d \times d$ matrix and c is a normalizing constant.

Assumption 1 is satisfied by multivariate normal or logistic distributions as well by multivariate versions of the Student's t-distribution, the Laplace distribution, and many others. The correspondence between Assumption 1 and the multivariate normal distribution is particularly important since the normal distribution arises frequently in relevant applications. For instance, if voters for beliefs about candidates by averaging over a number of independent characteristics of the election (e.g. demographic features of the district, events in national politics, changes in the economy) then the Central Limit Theorem implies that Assumption 1 should hold as the the number of relevant characteristics gets large.

A weaker assumption is needed for voter ideal points. Specifically, voter ideal points must be angularly symmetric. This concept is defined below:

Assumption 2. The distribution G of voter ideal points in *angularly symmetric* around the point $(0,\ldots,0)$ (Liu, 1988): any hyperplane passing through $(0,\ldots,0)$ divides $\mathbb{R}^d$ into two half spaces with equal probability under G.

An equivalent statement of Assumption 2 is that $\mathbf{z}_i/||\mathbf{z}_i||$ and $-\mathbf{z}_i/||\mathbf{z}_i||$ have the same distribution. Assumption 2 is weaker than Assumption 1 because all elliptical distributions are angularly symmetric but some angularly symmetric distributions are not elliptical.[13] A strong aspect of Assumptions 1 and 2 is that the voter and candidate distribution are symmetric about the *same* point. This would hold, for instance, in a model in which candidates are randomly drawn from the pool of voters.

To characterize the type of equilibrium described in Example 3, we need a general description of how candidates may rotate the issue dimensions in order to credibly reveal information. Let $\mathbf{Q}\Lambda\mathbf{Q}^{-1} = \Sigma$ be an eigendecomposition of the matrix Σ, where Λ is a diagonal matrix with the diagonal elements equal to the eigenvalues of Σ, and the columns of $\mathbf{Q}$ are the corresponding eigenvectors. This decomposition is useful because any two dimensions in the distribution of $\mathbf{Qz}$ are orthogonal. Furthermore, $\mathbf{Q}$ can be chosen to be a rotation matrix so that $\mathbf{Qz}$ can be interpreted as the coordinates of $\mathbf{z}$ when the axes of the space are rotated according to the eigenvalues of Σ. These new axes represent, in statistical parlance, the principal components of the distribution f and, in geometric terms, the principal axes of the ellipsoids formed by the contours of f.

I define the *principal orthant* of $\mathbf{z}$ as the set of points $\tilde{\mathbf{z}} \in \mathbb{R}^d$ such that $\mathbf{Qz}$ and $\mathbf{Q\tilde{z}}$ have the same sign on every dimension. There always exists an equilibrium in which the candidates reveal the principal orthant of their ideal points. Since the particular messages

[13] For an overview of these and other notions of multivariate symmetry in probability distributions, see Serfling (2004).

used by the candidates are unimportant, I restrict attention to messages consisting of a sequence numbers equal to -1 or 1. A principal orthant equilibrium is one in which $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \mathbf{m} \cdot \mathbf{z} >> 0\}$ if $\mathbf{m} \in \{-1, 1\}^d$ and $S(\mathbf{m}) = \emptyset$ otherwise.

Theorem 3. *There exists a principal orthant equilibrium under Assumptions 1 and 2.*

Theorem 3 implies that there is an equilibrium that partitions the policy space into 2^d convex sets. Since all ideal points along the same ray from the origin belong to the same principal orthant, principal orthant equilibria partially reveal the direction of each candidate from the center. Construction of a principal orthant equilibrium rests on the following property: given any pair of distinct messages in such a strategy profile, the sets of voter ideal points preferring the two candidates are divided by a hyperplane that passes through the origin of the policy space. By Assumption 2, any such distribution divides the voters' ideal point distribution in half.

Assuming the principal components of the ideal point distributions are well-understood by the voters, principal orthant equilibria are intuitive since they correspond to statements such as "I am socially conservative and economically liberal." In fact, since the first principal components explain the most variance in political preferences, there is reason to believe that these dimensions are the most likely to correspond to how voters think about politics. This argument is the motivation behind principal component analysis, factor analysis, and related methods which have been used to measure voters' political ideology in empirical work for decades (Schofield, Gallego and Jeon, 2011; Enelow and Hinich, 1984; Aldrich and McKelvey, 1977).

3.2.2 Full directional equilibria

As Example 4 demonstrated, there may also be a *full directional equilibrium* in which candidates fully reveal the direction of their ideal points from the center. In contrast to the principal orthant equilibria of Theorem 3, these equilibria exist only under relatively strong distributional assumptions. However, a discussion of full directional equilibria is valuable because it provides the starkest illustration of some principles of directional equilibria that are at work in the more general versions of the model. The full directional equilibria can be viewed as an outcome under the extreme case in which the candidates' ideal point distribution reaches perfect symmetry.

The distributional assumption required for candidate ideal points is spherical symmetry, which is defined below.

Assumption 3. The candidate distribution f is *spherically symmetric* around the point $(0,\ldots,0)$: $f(\mathbf{z}) = c \cdot f_0(\mathbf{z}'\Sigma^{-1}\mathbf{z})$ where f_0 is continuous and non-negative and the scale parameter Σ is a $d \times d$ identity matrix and c is a normalizing constant.

Assumption 3 is strictly stronger than Assumption 1: spherical symmetry is obtained by replacing the covariance matrix of an elliptical distribution with an identity matrix. For Assumption 3 to be satisfied there can be no correlation across dimensions and the variances on each dimension should be equal. Importantly, spherical symmetry also implies that the density along any ray away from the origin is the same. No additional assumptions are required for the distribution of voters' ideal points.

The *direction* of an ideal point $\mathbf{z} = (z^1, z^2, \ldots, z^d) \in \mathbb{R}^d$ is a vector of numbers $\phi(\mathbf{z}) = (\phi^1(\mathbf{z}), \ldots, \phi^{d-1}(\mathbf{z}))$ such that

$$z^i = ||\mathbf{z}|| \prod_{j<i} \sin \phi^j(\mathbf{z}) \cos \phi^i(\mathbf{z})$$

for all $i \in \{1, \ldots, d-1\}$ and

$$z^d = ||\mathbf{z}|| \prod_{j<d} \sin \phi^j(\mathbf{z}).$$

These are the angular components of the hyperspherical coordinates of $\mathbf{z}$. The function

$\phi(\mathbf{z})$ determines which point on the unit sphere lies on the same ray from the origin as the point $\mathbf{z}$.

A full directional equilibrium is one in which the candidates fully reveal the direction of their ideal points and nothing more. Since the specific messages chosen by the candidate are not of interest, it is convenient to consider candidate strategies that place positive probability only on messages on the unit hypersphere. Thus, a full directional equilibrium is one in which $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \phi(\mathbf{z}) = \phi(\mathbf{m})\}$ if $||m|| = 1$ and $S(\mathbf{m}) = \emptyset$ otherwise. In a directional communication equilibrium, voters' beliefs are concentrated on a ray starting from the origin and pointing in the direction of $\mathbf{m}$.

Theorem 4. *There exists a full directional equilibrium under Assumptions 2 and 3.*

Similar to the principal orthant equilibria in Theorem 3, construction of equilibria is driven by the fact that the supporters of each candidate are divided by a hyperplane passing through the origin. In a one-dimensional policy space, the equilibrium described in Theorem 4 corresponds to the equilibrium from Example 1 in which the candidates reveal whether their ideal point is to the left or to the right of the center. In multidimensional policy spaces, a directional communication equilibrium implies nearly complete information transmission in the sense that voters' beliefs are concentrated on very small sets. The interpretation of directional communication as being close to full revelation is supported by Corollary 1.

Corollary 1. *If $d > 1$ and Assumptions 2 and 3 hold then there exists and equilibrium in which voters' beliefs about candidates are concentrated on sets with measure zero containing the the candidates' true ideal points.*

Full directional equilibria clearly fall short of full revelation in other ways. Voters are unable to distinguish between moderate and extreme candidates. Therefore, there is a chance that voters will make mistakes even in this limiting case. However, the winner of the election will be the candidate who would move policy in the precise direction preferred by the majority of voters.

4 Discussion and Conclusions

An important line of research in political science asks under what conditions candidates can credibly reveal their policy intentions to voters. The most common understanding is that "cheap talk" campaigns should be completely uninformative and therefore repeated elections, informative party labels, or some other institution is needed to allow voters any opportunity to be informed. Using a formal model of electoral competition under incomplete information, I show that such institutions are not a necessary condition for campaigns to transmit information to voters. Instead, candidates will credibly reveal directional information about their policy preferences even in one-shot elections when talk is cheap.

The model assumes that candidates' *and* voters' ideal points are private information. This means that candidates are uncertain about which message would win the election. Though candidates believe that each message gives them an equal probability of victory before learning the voters' ideal points, the actual winner will be the candidate who offers the best expected payoff to a majority of voters given their realized ideal points. In this way, informative campaign communication increases the interim payoffs of the voters. Though uncertainty about voter ideal points is critical for this welfare implication, the assumption is irrelevant for existence of informative equilibria. For instance, if the electorate is replaced with a single voter at a fixed ideal point, there exist informative equilibria in which the voter is always indifferent between candidates and her welfare is not improved by informative campaign communications. Since candidates are unlikely to be completely certain about the ideal points of all voters, the results imply that electoral cheap talk can most likely improve voters' welfare.

Though the results provide strong examples of how political scientists' current understanding of cheap talk in elections is incomplete, further research is needed in order to fully understand the applicability of these models in empirical settings. Future research should explore the extent to which credible cheap talk is possible when candidates are not treated symmetrically (e.g. when one candidate is believed *ex ante* to be more extreme than the other). Furthermore, future research should assess the robustness of principal orthant and full directional equilibria to violations of the distributional assumptions used for their construction.

There is strong empirical evidence that politicians' communications to constituents are good predictors of their policy priorities in office (Sulkin, 2011; Grimmer, 2013). Though this finding is generally taken as evidence that reelection pressures bind candidates to tell the truth[14] or that voters nonstrategic,[15] this study shows that such findings are easily rationalized in some single-shot elections where lying is costless and voters are perfectly rational.

One proposed remedy to the supposed failure of cheap talk campaigns is informative party labels (Snyder and Ting, 2002; Ashworth and Bueno de Mesquita, 2008). In theories of informative parties, party leaders engage in screening activity to make adopting the party label a costly signal for candidates. This study demonstrates similar party labels could emerge even if parties engage in no screening and labels are pure cheap talk. In fact, the binary announcement equilibria of Theorems 1 and 2 could easily be interpreted as information conveyed by party labels. Furthermore, under common assumptions, candidates may reveal much more information than is captured by party labels (such as the orthants

[14]See Sulkin (2011), Chapter 2.

[15]In explaining his findings, Grimmer (2013) states: "While often useful for understanding bargaining, cheap talk models are unlikely to describe well how constituents access legislators' messages. Constituents...are unlikely to exert the cognitive effort to think strategically about how legislators are broadcasting information" (p.25).

of Theorem 3) so campaign speeches could continue to influence voters' beliefs even when party labels are informative.

One special case of the model also provides qualitative predictions about how candidates talk about policy in campaigns. Candidates reveal information about the direction of their preferences from the center without revealing whether their preferences are moderate or extreme in that direction. Furthermore, candidates reveal information about their preferences along orthogonal ideological dimensions that cut across issues. Thus, candidates and voters endogenously talk about issues in ideological terms that correspond well to political scientists' empirical notions of ideology.

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A Definition of $P(\cdot,\cdot)$

In the text, $P(S,S')$ is defined as the victory probability of a candidate revealed to be in the set $S \subseteq \mathbb{R}^d$ when the other candidate is revealed to be in the set $S' \subseteq \mathbb{R}^d$. For the sake of completeness, I will now fully define this probability and make a couple of observations about its properties.

For any $S \subseteq \mathbb{R}^d$ and $S' \subseteq \mathbb{R}^d$, let

$$T(S,S') = \left\{ \mathbf{z} \in \mathbb{R}^d : \int_S (\mathbf{z}-\mathbf{x})^d \frac{f(\mathbf{x})}{\int_S f(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}} d\mathbf{x} < \int_{S'} (\mathbf{z}-\mathbf{x})^d \frac{f(\mathbf{x})}{\int_{S'} f(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}} d\mathbf{x} \right\} \tag{1}$$

denote the set of voter ideal points at which a voter would prefer a candidate revealed to be in S to one revealed to be in S'. Thus, $G(T(S,S'))$ is the probability that a particular voter prefers a candidate revealed to be in S to one revealed to be in S'. Therefore, we have

$$P(S,S') = \sum_{\lceil \frac{n}{2} \rceil}^{n} \binom{n}{k} G(T(S,S'))^k \left(1 - G(T(S,S'))\right)^{n-k}. \tag{2}$$

Note that $P(S,S')$ is continuous with respect to the parameter $G(T(S,S'))$. Furthermore, since G is absolutely continuous, $P(S,S')$ is continuous with respect to changes in S and S'.

B Proofs of Results

Lemma 1. *If σ is such that $P(S(\mathbf{m}),S(\mathbf{m}')) = 1/2$ for all $\mathbf{m} \in \mathbb{R}^d$ and $\mathbf{m}' \in \mathbb{R}^d$ then σ is an equilibrium messaging strategy.*

Proof. Suppose that σ is such that $P(S(\mathbf{m}),S(\mathbf{m}')) = 1/2$ for all $\mathbf{m} \in \mathbb{R}^d$ and $\mathbf{m}' \in \mathbb{R}^d$. To show that σ is an equilibrium signaling strategy we will prove that both candidates are indifferent between all messages, which shows that they have no strict incentive to deviate from σ. Since the results should apply to games with sequential as well as simultaneous messages, we will consider two cases separately: one in which the candidate knows her opponent's message (such as when she is the second sender in the sequential game), and another in which she does not (such as when she is the first sender in the sequential game, or either sender in the simultaneous game). The proofs for both cases are below.

1. The expected utility to candidate $j \in \{A,B\}$ for sending any message $\mathbf{m} \in \mathbb{R}^d$ given that the other candidate $k =\in \{A,B\}\backslash\{j\}$ sends the message $\mathbf{m}'$ is

$$EU_j(\mathbf{m}|\mathbf{m}') \equiv P(S(\mathbf{m}),S(\mathbf{m}')) \cdot 0 - (1-P(S(\mathbf{m}),S(\mathbf{m}')))\mathbb{E}[||\mathbf{z}_j - \mathbf{z}_k||^2|\mathbf{m}'] \tag{3}$$

$$= -\frac{1}{2}\mathbb{E}[||\mathbf{z}_j - \mathbf{z}_k||^2|\mathbf{m}'] \tag{4}$$

since the candidate wins and implements her ideal point with probability $\frac{1}{2}$ (by our assumption on $P(\cdot)$), loses with probability $\frac{1}{2}$, and her beliefs about her opponents ideal point are derived from Bayesian updating on $\mathbf{m}'$. Since this payoff does not depend on $\mathbf{m}$, candidate j is indifferent over messages.

2. The total probability that a candidate $k \in \{A,B\}$ sends a message $\mathbf{m} \in \mathbb{R}^d$ is equal to $\Pr[\mathbf{m}_k = \mathbf{m}] = \int_{S(\mathbf{m})} f(\mathbf{z})d\mathbf{z}$. Let supp$(\sigma)$ denote the support of σ. The expected utility to candidate $j \in \{A,B\}$ for sending any message $\mathbf{m} \in \mathbb{R}^d$ given that she does not know the message chosen by her opponent $k =\in \{A,B\}\backslash\{j\}$ is

$$EU_j(\mathbf{m}) \equiv -\mathbb{E}[\mathbb{E}[||\mathbf{z}_j - \mathbf{z}_k||^2|\mathbf{m}']] \tag{5}$$

$$= -\frac{1}{2}\mathbb{E}[||\mathbf{z}_j - \mathbf{z}_k||^2] \tag{6}$$

by the law of total expectation and the fact that $P(S(\mathbf{m}),S(\mathbf{m}')) = 1/2$ for all $\mathbf{m} \in \mathbb{R}^d$ and $\mathbf{m}' \in \mathbb{R}^d$. Since this payoff does not depend on $\mathbf{m}$, candidate j is indifferent between messages.

Thus, in both cases any σ such that $P(S(\mathbf{m}), S(\mathbf{m}')) = 1/2$ for all $\mathbf{m} \in \mathbb{R}^d$ and $\mathbf{m}' \in \mathbb{R}^d$ is an equilibrium messaging strategy. ■

Theorem 1 *Let $d = 1$. Suppose there there exists a number $\tilde{z} > 0$ such that $P(\{z : z > \tilde{z}\}, \{z : z < \tilde{z}\}) < \frac{1}{2}$ and $P(\{z : z < -\tilde{z}\}, \{z : z > -\tilde{z}\}) < \frac{1}{2}$. There exists an informative equilibrium in which both candidates reveal whether their ideal points lie to the left or the right of some cutpoint.*

Proof. First consider condition (a). Let κ denote a cutoff strategy such that each candidate announces the message m_1 if $z_i \leq \kappa$ and m_2 if $z_i > \kappa$. For each $\kappa \in \mathbb{R}$, let $C^+(\kappa)$ be the set of ideal points with positive support in the candidate distributions that are weakly larger than κ and let $C^-(\kappa)$ bet the set of such points that are strictly less than κ. Since G and u are continuous, $P(C^-(\kappa), C^+(\kappa))$ is a continuous function of κ. By assumption, we have

$$P(C^-(-\tilde{z}), C^+(-\tilde{z})) < \frac{1}{2} < P(C^-(\hat{z}), C^+(\hat{z})).$$

By the intermediate value theorem, there must exist a point $z^* \in (-\tilde{z}, \hat{z})$ such that

$$P(C^-(z^*), C^+(z^*)) = P(C^+(z^*), C^-(z^*)) = \frac{1}{2}.$$

Thus, there exists a binary announcement informative equilibrium in which both candidates use a cutoff strategy of $\kappa = z^*$. ■

Theorem 2 *Let $d > 1$. There exists an informative equilibrium in which both candidates reveal whether their ideal points lie on one side or the other of some hyperplane.*

Proof. The proof closely follows that of Chakraborty and Harbaugh (2010). Consider a hyperplane passing through $(0, \ldots, 0)$ with orientation $\mathbf{s} \in \mathbb{S}^{d-1}$.[16] That hyperplane de-

[16]Note that $\mathbb{S}^{d-1}$ denotes the (d-1)-dimensional unit hypersphere.

noted $\mathbf{h}(\mathbf{s})$ and divides the space into two halfspaces, $H^+(\mathbf{s}) = \{\mathbf{z}\cdot\mathbf{h}(\mathbf{s}) > 0\}$ and $H_-(\mathbf{s}) = \{\mathbf{z}\cdot\mathbf{h}(\mathbf{s}) < 0\}$. If $P(H^+(\mathbf{s}), H_-(\mathbf{s})) = \frac{1}{2}$ then we are done. Otherwise, assume without loss of generality that $P(H^+(\mathbf{s}), H_-(\mathbf{s})) < \frac{1}{2}$. Since G and u are continuous, $P(H^+(\mathbf{s}), H_-(\mathbf{s}))$ is a continuous function of $\mathbf{s}$. Furthermore, for any two antipodal points $\mathbf{s}, -\mathbf{s} \in \mathbb{S}^{d-1}$, we have $H^+(\mathbf{s}) = H_-(-\mathbf{s})$ which implies that $P(H^+(\mathbf{s}), H_-(\mathbf{s})) = 1 - P(H^+(-\mathbf{s}), H_-(-\mathbf{s}))$. It follows that the map $\Delta(\mathbf{s}) = P(H^+(\mathbf{s}), H_-(\mathbf{s})) - P(H^+(-\mathbf{s}), H_-(-\mathbf{s}))$ is odd (i.e. $\Delta(\mathbf{s}) = -\Delta(-\mathbf{s})$ for all $\mathbf{s}$) and continuous in $\mathbf{s}$. The Borsuk-Ulam thoerem[17] implies that there exists some $\mathbf{s}^* \in \mathbb{S}^{d-1}$ such that $\Delta(\mathbf{s}^*) = 0$, which means that $P(H^+(\mathbf{s}^*), H_-(\mathbf{s}^*)) = P(H_-(\mathbf{s}^*), H^+(\mathbf{s}^*)) = \frac{1}{2}$. Thus, there exists a binary announcement informative equilibrium in which both candidates announce which of the half-spaces separated by $\mathbf{h}(\mathbf{s}^*)$ contains their ideal points. ■

Theorem 3 *There exists a principal orthant equilibrium under Assumptions 1 and 2.*

Proof. Let $\mathbf{y} = \mathbf{Qz} = (y^1, \ldots, y^d)$. Since Σ is a symmetric real matrix, $\mathbf{Q}$ is an orthogonal matrix, meaning that $\mathbf{Q}'\mathbf{Q} = \mathbf{I}_d$. Some algebraic manipulation shows that

$$f(\mathbf{y}) = f(\mathbf{Qz}) = c \cdot f_0(\mathbf{z}'\mathbf{Q}'\mathbf{Q}\Lambda^{-1}\mathbf{Q}^{-1}\mathbf{Qz}) = c \cdot f_0(\mathbf{z}'\Lambda^{-1}\mathbf{z}). \tag{7}$$

Since Λ^{-1} is a diagonal matrix, this implies that the dimensions of $\mathbf{Qz}$ are distributed independently and we can write

$$f(\mathbf{y}) = \prod_{j=1}^{d} f_j(y_j) \tag{8}$$

where f_j is the marginal density of y_j. Furthermore, since $f(\mathbf{y}) = f(-\mathbf{y})$, we have $\int_{-\infty}^{0} f_j(y_j)dy_j = \int_0^\infty f_j(y_j)dy_j = \frac{1}{2}$ and $\int_{-\infty}^{0} y_j^2 f_j(y_j)dy_j = \int_0^\infty y_j^2 f_j(y_j)dy_j$ for all j. The latter implies that $\mathbb{E}[||\mathbf{z}||^2|\mathbf{m}] = \mathbb{E}[||\mathbf{z}||^2|\tilde{\mathbf{m}}]$ and $\mathrm{Var}[\mathbf{z}|\mathbf{m}] = \mathrm{Var}[\mathbf{z}|\tilde{\mathbf{m}}]$ if $S(\mathbf{m})$ and $S(\tilde{\mathbf{m}})$ are both principal or-

[17] One statement of the Borsuk-Ulam theorem is as follows: For every odd mapping $g : \mathbb{S}^{d-1} \to \mathbb{R}^{d-1}$ there exists a point $\mathbf{s} \in \mathbb{S}^d$ satisfying $g(\mathbf{s}) = 0$. See for instance, Theorem 2.1.1 (BU1b) in Matousek (2007).

thants of $\mathbb{R}^d$ with respect to f.

Let $\mathbb{E}[||\mathbf{z}||^2|\mathbf{m}] = \bar{z}$ and $\text{Var}[\mathbf{z}|\mathbf{m}] = V$ and let $\bar{s}(\mathbf{m})$ and $\bar{s}(\tilde{\mathbf{m}})$ be the (vector-valued) means of $S(\mathbf{m})$ and $S(\tilde{\mathbf{m}})$, respectively. The mean-variance representation of quadratic expected utility implies that $\mathbb{E}[u(\mathbf{z}_A, \mathbf{z}_i)|\mathbf{m}_A = \mathbf{m}] > \mathbb{E}[u(\mathbf{z}_B, \mathbf{z}_i)|\mathbf{m}_B = \tilde{\mathbf{m}}]$ if and only if

$$-||\mathbf{z}_i - \bar{s}(\mathbf{m})|| - V > -||\mathbf{z}_i - \bar{s}(\tilde{\mathbf{m}})|| - V \tag{9}$$

$$-||\mathbf{z}_i|^2 - \bar{z}^2 + 2\mathbf{z}_i \cdot \bar{s}(\mathbf{m}) - V > -||\mathbf{z}_i|^2 - \bar{z}^2 + 2\mathbf{z}_i \cdot \bar{s}(\tilde{\mathbf{m}}) - V \tag{10}$$

$$2\mathbf{z}_i \cdot \bar{s}(\mathbf{m}) > 2\mathbf{z}_i \cdot \bar{s}(\tilde{\mathbf{m}}) \tag{11}$$

$$\mathbf{z}_i \cdot (\bar{s}(\mathbf{m}) - \bar{s}(\tilde{\mathbf{m}})) > 0. \tag{12}$$

The set of $\mathbf{z}_i$ satisfying this condition defines an open halfspace with $(0,\dots,0)$ on the boundary. Since G is angularly symmetric, this implies that each candidate expects to gain any voter's support with probability $\frac{1}{2}$. Since this holds for any pair of principal orthants, no candidate has a strict incentive to deviate from this strategy. ■

Theorem 4 *There exists a full directional equilibrium under Assumptions 2 and 3.*

Proof. If $\Sigma = k\mathbf{I}_d$ then $\mathbf{z}'\Sigma\mathbf{z} = k||\mathbf{z}||^2$ for all $\mathbf{z} \in \mathbb{R}^d$. Thus, we can write $f(\mathbf{z}) = f^*(||\mathbf{z}||)$ where the domain of $f^*(||\mathbf{z}||)$ is $\mathbb{R}_+$. For any $\mathbf{z}$ and $\tilde{\mathbf{z}}$, let $\theta(\mathbf{z}, \tilde{\mathbf{z}})$ be the angle enclosed by the two vectors.

By the law of cosines,

$$||\mathbf{z} - \tilde{\mathbf{z}}||^2 = ||\mathbf{z}||^2 + ||\tilde{\mathbf{z}}||^2 - 2||\mathbf{z}||||\tilde{\mathbf{z}}|| \cos\theta(\mathbf{z}, \tilde{\mathbf{z}}). \tag{13}$$

Thus, the expected utility to voter i from electing a candidate $j \in \{A, B\}$ using the full directional communication strategy who sends the message m is

$$-\mathbb{E}[||\mathbf{z}_i - \mathbf{z}||\ |m] = -\int_0^{\infty} \left[||\mathbf{z}||^2 + r^2 - 2||\mathbf{z}||r\cos\theta(\mathbf{z}_i,\mathbf{m})\right] dr \tag{14}$$

$$= -||\mathbf{z}_i||^2 - \int_0^{\infty} r^2 f^*(r)dr + 2||\mathbf{z}_i||\cos\theta(\mathbf{z}_i,\mathbf{m})\int_0^{\infty} r^2 f^*(r)dr. \tag{15}$$

Therefore, if A and B send $\mathbf{m}_A$ and $\mathbf{m}_B$ respectively and both use full directional communications strategies, i strictly prefers A to B if and only if

$$-||\mathbf{z}_i||^2 - \int_0^{\infty} r^2 f^*(r)dr + 2||\mathbf{z}_i||\cos\theta(\mathbf{z}_i,\mathbf{m}_A)\int_0^{\infty} r^2 f^*(r)dr \tag{16}$$

$$> -||\mathbf{z}_i||^2 - \int_0^{\infty} r^2 f^*(r)dr + 2||\mathbf{z}_i||\cos\theta(\mathbf{z}_i,\mathbf{m}_B)\int_0^{\infty} r^2 f^*(r)dr$$

$$2||\mathbf{z}_i||\cos\theta(\mathbf{z}_i,\mathbf{m}_A) > 2||\mathbf{z}_i||\cos\theta(\mathbf{z}_i,\mathbf{m}_B) \tag{17}$$

$$\cos\theta(\mathbf{z}_i,\mathbf{m}_A) > \cos\theta(\mathbf{z}_i,\mathbf{m}_B). \tag{18}$$

Since $\cos\theta(\mathbf{z}_i,\mathbf{m}) = \frac{\mathbf{z}_i\cdot\mathbf{m}}{||\mathbf{z}_i||||\mathbf{m}||} = \frac{\mathbf{z}_i\cdot\mathbf{m}}{||\mathbf{z}_i||}$ by the definition of the dot product and the fact that $||m|| = 1$, we have $\cos\theta(\mathbf{z}_i,\mathbf{m}_A) > \cos\theta(\mathbf{z}_i,\mathbf{m}_B)$ if and only if $\mathbf{z}_i \cdot (\mathbf{m}_A - \mathbf{m}_B) > 0$. Thus, the set of $\mathbf{z}_i$ satisfying this condition defines an open halfspace with $(0,\ldots,0)$ on the boundary. By the assumption that G is angularly symmetric, this implies that each voter strictly prefers A to B with probability $\frac{1}{2}$ and, by symmetry, strictly prefers B to A with the same probability. Furthermore, since this holds for any two messages, neither candidate has a strict incentive to deviate from the full directional communication strategy. ■