Informational Lobbying and Legislative Voting

Keith E. Schnakenberg [*]

Abstract

I analyze a model of interest group influence on legislative voting through information transmission. The model shows how interest groups may craft different messages to target different winning coalitions in order to influence the outcome. If access to legislators is costly then interest groups prefer to coordinate with allied legislators by providing them with information that helps them to persuade less sympathetic legislators. The model reconciles informational theories of lobbying with empirical evidence suggesting that interest groups predominantly lobby those who already agree with them. The model also makes new predictions about the welfare effects of interest group influence: from an *ex ante* perspective, informational lobbying negatively effects the welfare of legislators. The results highlight the need for more theories of persuasion that take collective choice institutions into account.

Words in text: 7916
Words in headers: 58
Words outside text (captions, etc.): 774
Number of math inlines: 402
Number of math displayed: 22
Total: $7916 + 58 + 774 + 402 + 22 =$ **9172**

[*]Assistant Professor of Public Policy, Martin School of Public Policy and Administration, University of Kentucky. Contact: keith.schnakenberg@gmail.com. The author would like to thank John Patty, Maggie Penn, William Minozzi, Randy Calvert, Justin Fox, Ian Turner, Michael Nelson, and seminar participants at Université du Québec à Montréal, Yale University, and Washington University in St. Louis.

Interest group influence is commonly seen as a consequence of strategic information transmission from lobbyists to legislators. Legislators lack information about the electoral or policy consequences of their decisions. Lobbyists acquire relevant information and use it to present biased advice to lawmakers. This informational view of interest group lobbying originated from classic accounts of interest group behavior[1] and continued through formal models of communication under asymmetric information.[2]

The most significant challenge to informational theories of lobbying comes from Hall and Deardorff (2006). The puzzle is the following: Informational theories of lobbying predict that interest groups will concentrate on lobbying the legislators that need persuading. In contrast, real lobbyists focus on legislators who already agree with them. Therefore, Hall and Deardorff conclude that most lobbying is not about changing legislators' minds. Instead, lobbying is *budget-based*: lobbyists subsidize the work of like-minded legislators in order to make them more effective champions of their mutual goals.

Should we conclude that lobbying has little to do with persuasion? I argue that this conclusion would be misguided. In fact, data from case studies, surveys, and in-depth interviews suggest that lobbyists commonly try to persuade lawmakers. Interest group scholars therefore face a challenge: we must develop theories of lobbying that can explain the prevalence of persuasive efforts by interest groups as well as which politicians they choose to lobby.

This study illustrates one way that informational accounts of lobbying can be reconciled with the empirical evidence. Many of the apparent limitations of informational lobbying theories do not stem from the idea of lobbying as persuasion but from other auxiliary simplifying assumptions. Most notably, most informational models involve only one legislator. This is limiting in two ways. First, interest groups do not lobby a single legislator in a vacuum but must instead build coalitions in favor of their preferred policy. Second, though one-legislator models depict the legislator as a passive recipient of information, legislators are active advocates and partners of interest groups.

[1] See, for instance, Truman (1953, p. 332-335), Bauer, Pool and Dexter (1972, Ch. 10-12), and Milbrath (1963, Ch. 9-13).

[2] These will be reviewed in more detail below. See Grossman and Helpman (2001) and Wright (1996) for detailed reviews of these models.

I consider a series of models that show how predictions about informational lobbying can be dramatically different for multimember legislatures. First, the conditions under which persuasion can be successful are considerably different in legislatures. Though it may seem obvious that persuading one member of Congress is easier than persuading 218, majority rule actually expands opportunities for persuasion. The mechanism exploits the fact that some information has asymmetric effects on legislators. Since the interest group only needs a winning coalition to support its preferred policy, its welfare is unchanged when legislators outside that winning coalition receive negative information about the policy. Therefore, an interest group can influence policies by designing its messaging strategy to target different winning coalitions at different times.

Informational lobbying to multimember legislatures also has significantly different policy implications compared to single legislator environments. Though information transmission to a rational decision-maker is always welfare enhancing, those welfare gains can be reversed in voting environments (Schnakenberg, 2015). In this model, interest group lobbying is bad for legislators' welfare: most legislators expect to be outside of the winning coalition often enough that they would prefer to bar the interest group from offering any advice.

I also show that informational lobbying may be most often directed at allies. This result rests on two facts: access to legislators is costly and legislators can actively lobby for their preferred position. Given this, interest groups prefer to gain access to allied legislators and provide them with information that helps them persuade opponents. The ultimate goal is persuasion of opponents through information transmission, but the mechanism is one in which lobbyists gain influence by supporting the efforts of like-minded legislators.

1 Existing Theories

Theories of lobbying propose a variety of mechanisms through which interest groups might influence policy. One such mechanism, typically postulated in models of vote-buying, is that interest groups trade resources such as campaign contributions for favorable votes (Grossman and Helpman, 1994, 2001). Vote-buying models motivated work seeking to explain how contributions influence policy despite the incentives of politicians to renege on agreements after contributions are made.[3] Furthermore, scholars have embedded vote-buying into settings with many legislators to analyze whether interest groups buy bare majorities or supermajorities[4], whom interest groups should lobby[5] and to analyze the effects of legislative institutions[6]. However, vote-buying models do not explain the majority of interest group behaviors. For example, lobbying expenditures greatly exceed campaign contributions for most interest groups[7] and lobbying expenditures are generally not direct transfers to politicians. Furthermore, lobbying firms exert more effort to hire policy experts, conduct policy research, and make substantive claims to politicians than would be expected from a pure vote-buying perspective. Thus, though vote-buying models have proven to be analytically useful, these models should be supplemented with explanations for how lobbyists substantively advocate for policy alternatives.

Another mechanism for interest group influence is informational: interest groups have policy information that legislators crave, and gain influence by strategically revealing that information to sway legislators toward their preferred policy. Early models of informational lobbying formalized this intuition. Potters and Van Winden (1990) demonstrated the effectiveness of informational political pressure in a dynamic game between an interest group and a legislator. Another two-player model by Potters and Van Winden (1992) showed that conflicts of interest between interest groups and legislators can prevent credible informational lobbying and that the imposition of lobbying costs restores credibility when conflicts of interest are not too severe.[8] Austen-Smith and Wright (1992) analyzed a model in which two opposing interest groups transmit information to a single legislator in order to influence her vote and showed that informational lobbying can improve the

[3]For instance, McCarty and Rothenberg (1996) suggest that commitment problems stand in the way of long-term relationships between donors and legislators, while Kroszner and Stratmann (2005) suggest that some of these commitment problems can be resolved by rewarding legislators for building a reputation for consistent policy support.

[4]For instance, Groseclose and Snyder (1996) show that interest groups are incentivized to buy coalitions larger than minimal winning coalitions when lobbying is competitive and interest groups move sequentially.

[5]Denzau and Munger (1986) show that interest groups ought to buy votes from legislators whose constituents are indifferent about the policy.

[6]Diermeier and Myerson (1999), for example, utilize a vote-buying model to show that bicameral legislatures have an incentive to create institutional hurdles within each chamber to maximize their monetary payoffs.

[7]Drutman (2015), in Figure 1.6 of his book, shows that corporate lobbying expenditures are, on average, 12 times higher than campaign contributions.

[8]This relates to a more general point, due to Crawford and Sobel (1982), that cheap talk is less informative the more the preferences of the sender and receiver diverge.

legislator's policy decisions.

Scholars quickly recognized a limitation of informational lobbying theories: they could not explain why interest groups predominately lobby policymakers who are already convinced of their positions.[9] Austen-Smith and Wright (1994) offered the first potential solution to this problem in the form of counteractive lobbying: interest groups may sometimes lobby allies in order to prevent them from being influenced by opposing lobbyists.[10] However, as Hall and Deardorff (2006) note, this theory does not fully account for the empirical patterns: the counteractive lobbying model predicts that "groups do lobby their allies, but they lobby only their weak allies, do so no more than their weak enemies, and do so less than undecided legislators" (p. 71).

An alternative theory, suggested by Bauer, Pool and Dexter (1972, p. 353) and more fully articulated in Hall and Deardorff (2006) is that interest group lobbyists provide labor and expertise to subsidize the work of legislators who already agree with them. In this theory, persuasion has little to do with most lobbying, since "the proximate objective of this strategy is *not* to change legislators' minds" (p. 69). Lobbying as persuasion is possible, according to Hall and Deardorff, but should be expected only under relatively rare circumstances.

Hall and Deardorff's explanation solves one puzzle but creates another: if lobbyists simply subsidize like-minded legislators, why do they waste so much time building a case for their policies? Surveys of interest group representatives seem to suggest that persuasion is an important component of influence. In Schlozman and Tierney's (1986) survey, for instance, the top three activities that consume time and resources for organizations are contacting government officials to present their point of view (35%), testifying at hearings (27%), and presenting research results (27%).[11]

[9] The fact that interest groups predominately lobby allies is well known. See, for example, Bauer, Pool and Dexter (1972, p. 353) and Hojnacki and Kimball (1998, 1999), though see Kollman (1997) for an alternative explanation for this fact.

[10] Vote buying models have also predicted that interest groups should lobby fence-sitters and moderate opponents. In that literature, Groseclose and Snyder's (1996) explanation for lobbying supermajorities is similar to Austen-Smith and Wright's (1994) model of counteractive lobbying. The intuition is that interest groups must expand the set of individuals they are willing to lobby when they face competition from other groups.

[11] This information is found in Table 7.2 (p. 151). Subsidy-like activities were also common in these data: Consulting with officials to plan legislative strategy (19%) was the seventh-ranked activity consuming time and resources, helping draft legislation (11%) was the eleventh-ranked activity, drafting regulations (7%) was sixteenth, contributing personnel to campaigns (2%) and doing favors for officials who need assistance (2%) also consumed time and resources for a small number of organizations.

More recent evidence paints a similar picture. In Baumgartner et al. (2009), 61% of organizations relied on disseminating in-house research to policymakers and 46% relied on disseminating external research, while only 39% helped draft legislative language.[12]

Prominent case studies also lend support to the idea that lobbyists engage in persuasion. For instance, Hansen (1991) argued that the farm lobby gained influence by providing information to legislators about how constituents would react to policy choices. Wright's (1996) analysis of lobbying efforts surrounding Robert Bork's nomination to the Supreme Court showed that lobbyists used a combination of persuasion and grassroots lobbying to achieve their goals (p. 97-103). In another example, Drutman and Hopkins (2013) analyzed the corpus of emails sent by Enron employees.[13] The emails revealed that Enron's lobbying efforts relied on using their "monopoly on policy-relevant information" to persuade legislators (p. 20). All of this evidence suggests that a significant proportion of lobbyists' time is spent on activities more clearly associated with informational lobbying. Current theories cannot simultaneously explain the prevalence of informational tactics and groups' predominant focus on allies.

2 Legislators and Interest Groups as Partners in Advocacy

One mechanism for persuasion by lobbyists, which has been noted but not explored systematically in previous theoretical models, is that lobbyists use their allies as intermediaries to persuade less sympathetic legislators. For instance, Austen-Smith and Wright (1994) recognize the possibility that respondents to their questionnaires may "consider lobbying to involve consultations with their legislative friends in order to have them indirectly lobby other less sympathetic legislators (p. 36)." Hall and Deardorff (2006) point to the same possibility as a way that their theory may complement informational lobbying theories, noting that "...[lobbyists] should also provide political intelli-

[12]This information comes from Table 8.1 of Baumgartner et al. (2009). There were some notable differences from Schlozman and Tierney's (1986) data: for instance, testifying at hearings was relatively less common (14.8%). As in Schlozman and Tierney's survey, personal contact with members of Congress was the most commonly used tactic (81%), though the question wording made it less clear whether personal contact was related to persuasion rather than conducting day-to-day business.

[13]Emails sent by Enron employees were acquired by the Federal Energy Regulatory Commission during its investigation of the company, and were later made public by researchers.

gence to their legislative allies so that the latter might employ preference-centered strategies with their uncommitted colleagues" (p. 79). Similarly, Ainsworth (1997) notes that "...when lobbying enterprises exist, lobbyists are most apt to concentrate their activities on their congressional allies, insuring that their allies have sufficient means to mobilize other legislators" (p. 526).

Qualitative data also document lobbyists using allies as intermediaries to persuade unsympathetic legislators. For instance, in a sweeping study of lobbyists involving over 300 in-depth interviews with policy advocates and structured data on over 2,000 advocates over a large number of issue areas, Mahoney and Baumgartner (2015) conclude that "Outsiders and insiders together decide who might be the most effective contact for a given target, what argument might be most compelling to that individual, and they all share information such as vote tallies." The authors cite examples from in-depth interviews, in which advocates indicated that they worked closely with legislative allies to discuss how to approach messaging to other legislators in order to broaden the coalition in favor of their side.

Though scholars of interest groups appear to share a casual understanding that lobbyists use allied legislators as intermediaries for persuasion, this phenomenon is mostly absent from formal theories of lobbying to date. One exception is a paper by Caillaud and Tirole (2007) which also explores strategies that can be used to persuade a voting body to pass a bill. In that model, the lobbyist has no private information but can offer legislators a report that allows them to determine relevant information for themselves. By targeting key legislators the lobbyist can sometimes engineer "persuasion cascades" in which bringing key members on board sways the opinions of others. The mechanism in this paper differs from Caillaud and Tirole, since persuasion cascades are driven not by communication but by correlation between players' payoffs that lead some voters to support a policy once they learn that another voter benefits.

3 A Theory of Informational Lobbying in a Legislature

The limitations of most informational lobbying models stem from two common assumptions: that the targets of interest groups' lobbying efforts can be reduced to a single representative policymaker and that role of the legislator is limited to receiving information and voting rather than actively lobbying. In this section I argue that relaxing both assumptions allows us to construct models of informational lobbying that are more in line with existing empirical findings.

3.1 Model One: Lobbying with Unlimited Access

I relax the two traditional assumptions one at a time in order to help build intuition. Thus, I begin by characterizing informational lobbying in an environment with multiple legislators but where an interest group has automatic access to all legislators. The interest group is the only player with private information and is able to communicate directly to every legislator, so the legislator advocacy is irrelevant.

3.1.1 Game play and payoffs

Consider a legislature N consisting of n legislators who must decide whether or not to pass a proposed policy. An interest group P represents proponents of the new policy and may lobby the legislators on behalf of its members. The interest group's method of lobbying is to strategically transmit information about the effects of the policy to the legislators.

The legislators are uncertain about some piece of information that affects whether or not they should vote for the proposal. For instance, the legislators may lack information about the effects of the proposed policy or about public opinion in their districts regarding the proposal (Truman, 1953). Following typical game-theoretic conventions, all of the unknown policy-relevant information is represented by a state of the world, labeled ω. Specifically, the state of the world is a vector of n zeros and ones, where element i is equal to one if legislator i ought to vote in favor of the proposal and zero otherwise. For instance, if $n = 3$, the state $\omega = (1,0,1)$ means that legislators 1 and 3 will receive higher utility if the proposal passes and legislator 2 will receive higher utility if the proposal fails. The set $\Omega = \{0,1\}^n$ represents all feasible states of the world.

Though the legislators are ignorant of the value of ω, they share common prior beliefs. This belief is represented by a function f, according to which each legislator $i \in N$ is associated with a probability $p_i \in (0,1)$ of benefiting from the proposal and these probabilities are independent across legislators. Formally, if $\mathbf{p} = (p_1, \ldots, p_n)$, the prior probability distribution for each $\omega = (\omega_1, \ldots, \omega_n) \in \Omega$ is equal to $f(\omega, \mathbf{p}) = \prod_{i \in N} p_i^{\omega_i}(1-p_i)^{1-\omega_i}$. The common prior is meant to represent all of the publicly available information that informs what legislators think about the policy prior to the introduction of any private information from interest groups.

The sequence of play is as follows. First, P observes the state of the world. This represents the idea that the interest group has obtained some private information – perhaps through private research or polling, or because it has special expertise on the policy issue – that it can use to try to persuade the legislators of its position. Second, P communicates to the legislators. Communication takes the form of a message m_P which is an element of the set Ω. For example, if $n = 3$, the message $m_P = (0,1,1)$ would be taken as a recommendation for legislators 2 and 3 to vote in favor of the proposal and for legislator 1 to vote against. The messages are cheap talk, meaning that their content does not directly affect the payoffs of the players. Finally, the legislators update their beliefs about the probability of benefiting from the policy and take an up-or-down vote over whether or not to implement the new policy. The legislature operates according to a q-majority rule, meaning that the proposal passes if and only if at least q legislators vote in favor, where $\frac{n}{2} < q < n$. The policy outcome is represented by the variable x, where $x = 1$ when the proposal passes and $x = 0$ when the proposal fails.

Each legislator $i \in N$ has preferences represented by the utility function

$$u_i(x, \omega) = \begin{cases} H & \text{if } x = 1 \text{ and } \omega_i = 1 \\ -L & \text{if } x = 1 \text{ and } \omega_i = 0 \\ 0 & \text{if } x = 0. \end{cases} \tag{1}$$

where H and L are both positive numbers. P's preferences are represented by the utility function $u_P(x) = x$. In other words, P always prefers that the legislature vote to pass the proposal, regardless of the state of the world.

3.1.2 Strategies and equilibrium

The players' strategies are plans describing how they will play the game at every information set. P's strategy, denoted σ_P, prescribes a probability distribution over possible messages for each possible state of the world. Thus, $\sigma_P(m|\omega)$ is the probability that P sends the message m after learning that the state of the world is equal to ω. For each $i \in N$, legislator i's strategy is a function v_i describing how that legislator will vote following every possible message by P.[14] The entire profile of legislators' voting strategies is denoted $v = (v_1, \ldots, v_n)$.

In addition to the players' strategies, the equilibrium predictions depend on the beliefs of the legislators. Though the legislators' prior beliefs are part of the definition of the game, the analysis also relies on the legislators' conditional beliefs following communication with the interest group. Let $\pi_i(m)$ denote the legislators' beliefs about the probability that $\omega_i = 1$ given that P has sent the message m.[15] Legislators' conditional beliefs are assumed to be correct – that is, consistent with Bayesian updating – following all messages. This ensures that interest group influence results from structural aspects of legislative policymaking rather than from psychological biases that make legislators easily manipulable.

The analysis characterizes perfect Bayesian equilibria in weakly undominated strategies. An equilibrium to the game is a profile of strategies such that: (a) P's messaging choices maximize its expected payoff given the legislators' strategies, (b) legislators vote in favor of the proposal if and only if $\pi_i(m)H - (1-\pi_i(m))L \geq 0$, and (c) $\pi_i(m)$ is never inconsistent with Bayesian updating given P's strategy.

Like all cheap talk games, this game admits multiple equilibria some of which involve no information transmission at all. Unless no other equilibria exist, uninformative equilibria are implausible: any interest group should fire a lobbyist who babbles uninformatively when it is possible

[14] I allow the interest group to play a mixed strategy but limit the legislators to pure strategies. When it is relevant, I assume that legislators vote in favor of the proposal when they are indifferent, though relaxing this assumption would affect the results only in knife-edge cases.

[15] $\pi_i(m)$ is not a full description of a legislators' posterior beliefs, but only the decision-relevant component of those beliefs. Let $\Delta(\Omega)$ denote the set of all probability distributions over Ω. The full posterior belief is a function $g : \Omega \to \Delta(\Omega)$ giving the probability of each $\omega \in \Omega$ following any message. Then $\pi_i(m) = \sum_{\omega:\omega_i=1} g(\omega|m)$. This formulation is used in the appendix but I focus on $\pi_i(m)$ in the main text to simplify the presentation.

to influence outcomes instead. Therefore, I will focus on equilibria that are optimal from the perspective of the interest group.

3.1.3 Analysis

My analysis of the model of lobbying with unlimited access seeks answers to three questions: When is it possible for the interest group to influence the outcome? When influence is possible at all, what kinds of messaging strategies help the interest group to be influential? Finally, what are the implications of interest group influence for legislators' welfare? Proofs of all results are contained in the appendix.

An *influential* equilibrium is one in which a proposal that would have failed passes with some positive probability due to P's lobbying efforts. Though this definition allows for different outcomes following different states or messages, the assumptions of the model imply a stronger notion of influence: if P influences policy at all, the proposal must pass with probability one. The reason is that P cannot commit to messaging strategies that cause passage following some messages and failure following others. P would deviate from such a strategy by only sending successful messages.

Lemma 1. *In any influential equilibrium, the proposal passes with probability one.*

It follows from Lemma 1 that if the legislature were replaced with a single policymaker then it would be impossible for P to influence the outcome: Suppose that P could influence this single policymaker ($N = \{1\}$) whose prior expected utility from passing the proposal is negative. Lemma 1 tells us that the policymaker must always approve the proposal, which means that $\pi_1(m) > p_1$ following every speech m by P. However, the law of total probability tells use that $\sum_{m \in \Omega} \pi_1(m) \Pr[m_p = m] = p_1$, so it can never be true that every posterior belief is more optimistic than the prior.

Why should influence work differently when the target is a legislature rather than a single policymaker? To see the difference, consider an example involving a majority rule committee of three legislators.

Example 1. Let $n = 3$. The committee uses a simple majority rule ($q = 2$). Suppose that a proposed project costs \$200 per district and each legislator believes that the project would yield her district a benefit worth \$600 with probability $\frac{1}{4}$ (in terms of the model setup, we have $L = 200$, $H = 600 - 200 = 400$, and $p_1 = p_2 = p_3 = \frac{1}{4}$). Without any lobbying, the proposal would fail in a unanimous vote, since every legislators' expected benefit is $-\$50$. A legislator should support the proposal only if the posterior belief that her district will benefit is at least $\frac{1}{3}$.

Consider the following strategy for P. After observing which legislators benefit from the project, P will choose a minimal winning coalition of two legislators and recommend that they vote in favor. P chooses among the three minimal winning coalitions in the following way: If no legislators benefit from the project, P chooses a random coalition. If one legislator benefits from the project, P randomly chooses between the two coalitions that include the true beneficiary (e.g. if $\omega = (1,0,0)$ then P will choose randomly between $(1,1,0)$ and $(1,0,1)$). If exactly two legislators benefit from the project P chooses the coalition consisting of both true beneficiaries with probability one. If all legislators benefit from the project, P chooses a random coalition.

Table 1 depicts the signaling strategy and the prior and posterior probability calculations for this example. For the two legislators in the chosen coalition, the posterior probability of benefiting is now $\frac{47}{128}$ which is greater than $\frac{1}{3}$, so two legislators will always vote "yes." The posterior probability of benefiting for the remaining legislator is $\frac{1}{64}$ (i.e. the probability of three beneficiaries) so one legislator will always vote "no." ■

Example 1 shows that a lobbyist may persuade a majority of legislators to support her position even when none of the legislators would have been persuadable on their own. The lobbyist persuades the legislators by targeting different winning coalitions in different states of the world. Following each message, members of the targeted coalition learn that they have a higher probability of benefiting from the proposal and vote in favor of passage. The remaining members of the legislature learn that they are even less likely to benefit from the proposal and are even more strongly opposed than prior to P's lobbying efforts, but this does not affect the payoff of P, who is only interested in whether or not the proposal passes.

The next results establish general conditions under which P may influence the legislature.

ω	$\sigma_P((1,1,0)\|\omega)$	$\sigma_P((1,0,1)\|\omega)$	$\sigma_P((0,1,1)\|\omega)$	$f(\omega,\mathbf{p})$	$\Pr[\omega\|(1,1,0)]$	$\Pr[\omega\|(1,0,1)]$	$\Pr[\omega\|(0,1,1)]$
$(0,0,0)$	$^1/_3$	$^1/_3$	$^1/_3$	$^{27}/_{64}$	$^{27}/_{64}$	$^{27}/_{64}$	$^{27}/_{64}$
$(0,0,1)$	0	$^1/_2$	$^1/_2$	$^9/_{64}$	0	$^{27}/_{128}$	$^{27}/_{128}$
$(0,1,0)$	$^1/_2$	0	$^1/_2$	$^9/_{64}$	$^{27}/_{128}$	0	$^{27}/_{128}$
$(0,1,1)$	0	0	1	$^3/_{64}$	0	0	$^9/_{64}$
$(1,0,0)$	$^1/_2$	$^1/_2$	0	$^9/_{64}$	$^{27}/_{128}$	$^{27}/_{128}$	0
$(1,0,1)$	0	1	0	$^3/_{64}$	0	$^9/_{64}$	0
$(1,1,0)$	1	0	0	$^3/_{64}$	$^9/_{64}$	0	0
$(1,1,1)$	$^1/_3$	$^1/_3$	$^1/_3$	$^1/_{64}$	$^1/_{64}$	$^1/_{64}$	$^1/_{64}$

	$m_P = (1,1,0)$	$m_P = (1,0,1)$	$m_P = (0,1,1)$
$\pi_1(m_P)$:	$\mathbf{^{47}/_{128}}$	$\mathbf{^{47}/_{128}}$	$^1/_{64}$
$\pi_2(m_P)$:	$\mathbf{^{47}/_{128}}$	$^1/_{64}$	$\mathbf{^{47}/_{128}}$
$\pi_3(m_P)$:	$^1/_{64}$	$\mathbf{^{47}/_{128}}$	$\mathbf{^{47}/_{128}}$

Table 1: Calculations for Example 1. In the top half of the table: The first column lists each possible state of the world. The second, third and fourth columns show the probabilities of the messages $(1,1,0)$, $(1,0,1)$ and $(0,1,1)$ (respectively) given each state of the world. The fifth column shows the prior probability of each state of the world. The sixth, seventh, and eighth columns show the posterior probability of each state of the world given the messages $(1,1,0)$, $(1,0,1)$, and $(0,1,1)$ (respectively). Since the total probability of each message is $^1/_3$, each posterior following each message M is $3 \cdot (\sigma_p(m_P|\omega) f(\omega,\mathbf{p})$. The bottom half of the table shows each legislators' posterior probability of benefiting form the policy following each message, calculated for each i by adding up the posterior probabilities of all states for which $\omega_i = 1$. Posterior probabilities that lead the legislator to vote in favor of passage are highlighted in **bold**.

Lemma 2 provides a necessary and sufficient condition for existence of an influential equilibrium. Proposition 1 provides a comparative statics result and a more easily interpretable sufficient (but not necessary) condition for interest group influence.

The set of posterior distributions for which the proposal passes is the set of all distributions over $\{0,1\}^n$ such that at least q voters have a probability of at least $\frac{L}{H+L}$ of benefiting from the policy.[16] That is, the *approval set* is:

$$W_q = \left\{ g \in \Delta(\{0,1\}^n) : \left| \left\{ i \in N : \mathbb{E}_g[\omega_i] \geq \frac{L}{H+L} \right\} \right| \geq q \right\} \tag{2}$$

where $\Delta(S)$ denotes the set of all probability distributions over any set S. Note that elements of W_q need not be distributions that can be expressed as n independent probabilities, so each $g \in W_k$ is a vector of probabilities over all 2^n different combinations of winners. Let $\mathrm{co}(W_q)$ denote the

[16] The equilibrium conditions state that $v_i =$ Yes if and only if $\pi_i(m)H - (1-\pi_i(m))L \geq 0$. It follows from simple algebraic manipulations that $v_i =$ Yes if and only if $\pi_i(m) \geq \frac{L}{L+H}$.

convex hull of W_q, that is, the set of all probability distributions that can be expressed as convex combinations of distributions in W_q.

Lemma 2. *There is an influential equilibrium in Model 1 if and only if* $f(\omega, \mathbf{p}) \notin W_q$ *and* $f(\omega, \mathbf{p}) \in co(W_q)$.

The argument for Lemma 2 characterizes the possible messaging strategies in terms of the posterior beliefs generated by each message in its support. By Lemma 1, proving existence of an influential equilibrium requires verifying that some messaging strategy leads to passage with probability one. In other words, we must verify the existence of some messaging strategy such that every message sent by P leads to a posterior belief in W_q. Fortunately, previous work shows that a set of posterior beliefs can be generated by realizations of a signal if and only if the prior distribution is in their convex hull (Kamenica and Gentzkow, 2011). Thus, an influential equilibrium can be supported if and only if the prior distribution is in the convex hull of the approval set.[17]

Lemma 2 is useful but does not easily lend itself to empirical predictions. Fortunately, Lemma 2 gives rise to more easily interpretable results that relate the existence of influential equilibria to the legislators' prior probabilities of benefiting from the policy. Proposition 1 has two parts. The first part shows that influential equilibria are monotonic with respect to the prior distribution: if P can influence the legislature under one prior distribution, then P can still guarantee passage if we increase some legislators' probabilities of benefiting and leave the rest of the game unaltered. The second part of Proposition 1 shows that one sufficient condition for influence can be stated in terms of a simple cutoff, labeled p^*. If all legislators' prior probabilities of benefiting are above p^*, then there exists an equilibrium in which the proposal always passes. Importantly, p^* is strictly lower than the belief at which legislators would support passage without lobbying, so the cutoff result shows again that P can persuade the legislature to pass a proposal that otherwise would have failed.

[17] Kamenica and Gentzkow (2011) analyze a model of *information control* in which the sender does not have private information but can design the content of a public signal. Alonzo and Câmara (2015) apply the same information control framework to a voting setting. A cheap talk model can be seen as a constraint on the information control model that limits attention only to signals that can be credibly disclosed when the sender knows the state of the world. Thus, Lemma 2 could be attained by applying Alonzo and Câmara's (2015) results and adding the requirement that the proposal passes with probability one. Geometrically, the result relies on the fact that W_q is not a convex set.

Proposition 1. *In Model 1:*

1. *Let* $\hat{\mathbf{p}} \geq \mathbf{p}$ *and assume* $f(\omega,\hat{p}) \notin W_q$*. If there is an influential equilibrium when prior beliefs are* $f(\omega,\mathbf{p})$ *then there is also an influential equilibrium when prior beliefs are* $f(\omega,\hat{\mathbf{p}})$*, holding everything else equal.*

2. *There exists* $p^* \in \left(0, \frac{L}{H+L}\right)$ *such that if* $p_i \geq p^*$ *for all* $i \in N$ *then there exists an equilibrium in which the proposal always passes.*

The proof of Proposition 1 involves two steps. First, to prove part (1) I demonstrate that if the prior distribution $f(\omega,\mathbf{p})$ is in the convex hull of W_q then so is any distribution $f(\omega,\hat{p})$ with $\hat{\mathbf{p}} \geq \mathbf{p}$, which implies (by Lemma 2) that an influential equilibrium still exists for the increased prior distribution. Second, to prove part (2) I first assume that $p_i = p_j = \overline{p}$ for all i and j in N and derive the minimum value p^* of $\overline{p}$ for which there exists an influential equilibrium. Part (1) of the Proposition implies that there is also an equilibrium when legislators have different probabilities of benefiting but all probabilities are weakly greater than p^*.

Corollary 1 documents three substantive points about legislator welfare that follow from the previous results. First, any influential equilibrium reduces the *ex ante* expected utility of at least $n-q+1$ legislators. This point follows directly from Lemma 1. Since the proposal must pass with probability one in an influential equilibrium, the *ex ante* expected utility of each legislator is equal to her expected benefit from passing the proposal. Since it is assumed that the proposal would have failed in the absense of lobbying and therefore that P's influence was necessary for passage, this must mean that fewer than q legislators prefer informational lobbying over no lobbying.

Second, there are some parameters under which an influential equilibrium could unanimously reduce the welfare of the legislators. This point follows from the second part of Proposition 1. Since there is an influential equilibrium if $p_i \geq p^*$ for all $i \in N$ and p^* is strictly lower than the probability that would lead a legislator to support passage *ex ante*, an influential equilibrium might exists when no legislators would support passage. In these cases, influential equilibria are bad for the welfare of all legislators. Though the first welfare result is true of all influential equilibria in this paper, unanimously harmful lobbying will be much more difficult to support when access to legislators is costly.

Third, influence is more difficult for higher supermajority rules. This follows immediately from the definition of the equilibria but is substantively useful because it relates to institutional arguments made elsewhere. In fact, a common argument in favor of supermajority rules is that they help prevent capture by special interests (McGinnis and Rappaport, 1998).

Corollary 1. *The following facts hold for Model 1:*

1. *Any influential equilibrium reduces the* ex ante *expected utility of at least $n-q+1$ legislators compared to the outcome with no information transmission*

2. *There exist parameters under which an influential equilibrium reduces the* ex ante *expected utility of all legislators compared to the outcome with no information transmission*

3. *Let $q'' > q'$. If there is an influential equilibrium when $q = q''$, the interest group can also guarantee passage when $q = q'$.*

Finally, Lemma 3 sets the stage for the analysis of the game with costly access. Part (3) shows that, if P has at least one ally in the legislature (that is, a legislator who would have approved the proposal without any lobbying) and there is an influential equilibrium to the game, then there is an outcome-equivalent influential equilibrium in which some ally learns nothing about her probability of benefiting from the proposal.

Lemma 3. *Assume that $p_i \geq L/(H+L)$ for some i. Then any influential equilibrium is outcome-equivalent to one in which $\pi_i(m) = p_i$ for all $m \in \Omega$.*

Lemma 3 results from the assumption that the prior probabilities of benefits are independent. Since the ally legislator's benefit does not give any information to the other legislators about their probability of benefiting, P can eliminate any information about the ally's benefit without changing the behavior of the remaining legislators.

3.2 Model Two: Lobbying with Limited Access

The preceding model clarifies how influencing a legislature differs from influencing an individual policymaker and spells out the machinery of building coalitions through information transmission. By designing many messages that target different winning coalitions, a lobbyist can influence a legislature when influencing a unitary policymaker would have been impossible. However, since the model involves only public messages sent to all legislators, it does not enable us to make predictions about who each interest group should lobby. In this section, I expand the model to include situations in which gaining access to individual legislators is costly. Since lobbying with limited access may create information asymmetries between legislators, the expanded model also allows legislators to actively lobby by passing information along to their peers.

3.2.1 Game play and payoffs

The game with limited access to legislators involves two additional moves. First, before P learns the state of the world, it must decide whether to invest in access to each legislator. Gaining access to legislators is costly – either in time and effort or in campaign contributions – so P prefers to gain access to the minimum number of legislators required to influence policy. The communication stage now differs from the previous model in that P's messages are perceived only by those legislators to which it has gained access. Second, after P's communication effort but prior to voting, legislators are recognized to speak publicly. Since only the legislators lobbied by P possess information that is unavailable to other legislators, we will focus only on those legislators' speeches and they will choose between passing on P's message or simply remaining silent.

The full sequence of play is as follows. First, P chooses whether or not to invest in access to each legislator. Let A_P denote the set of legislators to whom P has access. Second, P observes the state of the world and chooses a cheap talk message $m_P \in \Omega$ which is revealed only to legislators in A_P. Third, legislators in A_P are recognized to speak at which point they may either reveal m_P to their fellow legislators ($\mu_i = 1$) or reveal no information ($\mu_i = 0$).[18] Finally, legislators update

[18] Allowing partial revelation of information by legislators would not affect the outcomes of the model, since the message simply determines whether the proposal passes or fails.

their beliefs and vote as in Model 1.

Legislators' utility functions are the same as in Model 1. P has the same preferences over policies but now pays a cost for gaining access to legislators. P's preferences are represented by the utility function $u_P(x, A_P) = x - c|A_P|$ where $|A_P|$ is the total number of legislators in A_P and $c \in [0,1)$ is the per-legislator cost of access. When $c = 0$, P has unlimited access to the legislature and legislators' speeches are irrelevant, so Model 1 can be recovered as a special case of this game.

The addition of costly access to the model aligns this study with other models of informational lobbying with costly access (Austen-Smith, 1995, 1998; Cotton, 2015, 2012, 2009). In those models, contributions may enhance the credibility of information transmission, either about policy or about interest group preferences. In contrast, interest groups in this model make contributions prior to learning the state of the world, so contributions do not provide additional information beyond what would be expected in cheap talk models. Substantively, this assumption is justified by conceptualizing access to legislators as a long-term decision rather than as a response to a particular policy.

3.2.2 Strategies and equilibrium

P's strategy in Model 2 has two elements: a set A_P of legislators to which P will invest in access, and a messaging strategy σ_P which now prescribes a probability distribution over possible messages for each possible state of the world and each possible $A_P \subseteq N$. Thus, $\sigma_P(m|\omega, A_P)$ is the probability of sending the message m given that the state is revealed to be ω and that P has gained access to the set A_P of legislators.

Each legislator's strategy also has two components. Legislator i's speech strategy is a set $M_i(A)$ such that $\mu_i = 1$ when $m_P \in M_i(A)$ and $i \in A_P = A$. Finally, letting I_i denote all information available to legislator i at the time of voting,[19] legislator i's voting strategy is a function $v_i(I_i)$ describing how that legislator will vote at any information set.

An equilibrium is a strategy profile such that (a) P's lobbying and messaging choices maximize its expected payoff given the legislators' strategies, (b) each legislator's communication strategy

[19] Specifically, I_i includes A_P and m_P if $i \in A_P$ and $\{\mu_j\}_{j \in A_P}$ otherwise.

maximizes her expected payoff given the strategies of the other legislators, (c) legislators vote in favor of the proposal if and only if $\pi_i(I_i)H - (1 - \pi_i(I_i))L \geq 0$ and (d) $\pi_i(I_i)$ is never inconsistent with Bayesian updating using the other players' strategies.

3.2.3 Analysis

The analysis of Model 2 shows how interest groups may use allies as intermediaries to persuade less sympathetic legislators. Equilibria involving intermediaries share characteristics in common with previous informational and budget-based theories of lobbying. As in other informational theories of lobbying, the goal is to persuade legislators to change their vote by strategically transmitting information. However, as in budget-based theories, interest groups target allies for lobbying to help them achieve their mutual goals. It is important to make the distinction between the targets of lobbying activity and the targets of persuasion: interest groups focus on lobbying allies, but they do so to help persuade their opponents.

To illustrate how an interest group may use allies as intermediaries for informational lobbying, consider another three-legislator example.

Example 2. As in Example 1, there is a committee of $n = 3$ legislators that makes decisions according to a simple majority rule ($q = 2$). Also as in Example 1 the proposed project costs \$200 per district and yields a benefit for \$600 to some number of districts. In this example, however, the prior probabilities of benefiting are assymetric: Legislator 1 receives the benefit with probability $p_1 = {}^9/_{10}$ and the other two legislators receive the benefit with probability one fourth ($p_2 = p_3 = {}^1/_4$). As before, each legislator should support the proposal only if her probability of benefiting is at least ${}^1/_3$. Without any lobbying, legislators 2 and 3 would vote against the proposal and legislator 1 would vote in favor, leading to a failure of the proposal. In this example, lobbying is costly for P who pays a cost $c \in (0, 1)$ for every legislator to which it buys access.

Consider the following strategy. First, P will buy access only to legislator 1. In its communication with legislator 1, P will choose between the two messages $\{1, 2\}$ and $\{1, 3\}$ which we will interpret as the set of legislators to which P recommends voting in favor of the proposal. P chooses between these two messages as follows: If both 2 and 3 benefit from the proposal or if neither benefit, P will randomize between the two messages. If legislator 2 benefits but not legislator 3, P sends the message $\{1,2\}$ with probability one. If 3 benefits but not 2, P sends the message $\{1,3\}$ with probability 1.

ω	$\sigma_P(\{1,2\}\|\omega)$	$\sigma_P(\{1,3\}\|\omega)$	$f(\omega,\mathbf{p})$	$\Pr[\omega\|\{1,2\}]$	$\Pr[\omega\|\{1,3\}]$
$\{0,0,0\}$	$^1/_2$	$^1/_2$	$^9/_{160}$	$^9/_{160}$	$^9/_{160}$
$\{0,0,1\}$	0	1	$^3/_{160}$	0	$^3/_{80}$
$\{0,1,0\}$	1	0	$^3/_{160}$	$^3/_{80}$	0
$\{0,1,1\}$	$^1/_2$	$^1/_2$	$^1/_{160}$	$^1/_{160}$	$^1/_{160}$
$\{1,0,0\}$	$^1/_2$	$^1/_2$	$^{81}/_{160}$	$^{81}/_{160}$	$^{81}/_{160}$
$\{1,0,1\}$	0	1	$^{27}/_{160}$	0	$^{27}/_{80}$
$\{1,1,0\}$	1	0	$^{27}/_{160}$	$^{27}/_{80}$	0
$\{1,1,1\}$	$^1/_2$	$^1/_2$	$^9/_{160}$	$^9/_{160}$	$^9/_{160}$
				$M=\{1,2\}$	$M=\{1,3\}$
			$\pi_1(M)$:	$\mathbf{^9/_{10}}$	$\mathbf{^9/_{10}}$
			$\pi_2(M)$:	$\mathbf{^7/_{16}}$	$^1/_{16}$
			$\pi_3(M)$:	$^1/_{16}$	$\mathbf{^7/_{16}}$

Table 2: Calculations for Example 2. In the top half of the table: The first column lists each possible state of the world. The second and third columns show the probabilities of the messages $\{1,2\}$ and $\{1,3\}$ (respectively) given each state of the world. The fourth column shows the prior probability of each state of the world. The fifth and sixth columns show the posterior probability of each state of the world given the messages $\{1,2\}$ and $\{1,3\}$ (respectively). Since the total probability of each message is $^1/_2$, each posterior following each message M is $2\cdot(\sigma_p(M|\omega)f(\omega,\mathbf{p})$. The bottom half of the table shows each legislators' posterior probability of benefiting form the policy following each message, calculated for each i by adding up the posterior probabilities of all states for which $\omega_i=1$. Posterior probabilities that lead the legislator to vote in favor of passage are highlighted in **bold**.

For this strategy to be effective, it must always be incentive-compatible for legislator 1 to reveal the message to her fellow legislators and for two legislators to vote in favor of the proposal following either message. Table 2 summarizes the probability calculations for this strategy. Legislator 1 learns nothing from P about her own probability of benefiting from the proposal. However, the information she learns about the other legislators' benefits is enough to cause the proposal to pass. Since she already supports passage, Legislator 1 has a strict incentive to reveal the message to her fellow legislators. Once she does, the posterior probability of benefiting is raised to $^7/_{16}$ for one legislator and reduced to $^1/_{16}$ for another. As a result, the proposal passes following both messages. ■

Example 2 shows that an interest group may influence a legislature by buying access to one legislator who acts as an intermediary to other legislators. In the equilibrium in Example 2, the intermediary is an ally to the interest group. Furthermore, the information provided is never informative to the intermediary about whether or not she should vote to pass the proposal – instead, the targets of the interest group's messages are the legislators who are never directly lobbied. Proposition 2 establishes that these properties always hold for P's optimal equilibrium. The proof of Proposition 2 also provides a full description of the equilibrium including beliefs and actions off the equilibrium path.

Proposition 2. *Assume that $c > 0$ and that P has at least one ally (that is, $p_i > L/(H+L)$ for some i in N). Then P's optimal influential equilibrium has the following properties:*

1. *P lobbies exactly one legislator and that legislator is an ally, and*
2. *The lobbied legislator always conveys P's message to the legislature and always vote in favor of passage.*

The fact that an interest group only lobbies allies follows from the assumption that publicizing the group's information is optional for the lobbied legislator. This means that the chosen legislator must prefer passage of the proposal following every message. Since each player's expected benefit in the influential equilibrium must be equal to their prior expected benefit from passage, such an arrangement is only feasible when the legislator is already an ally.

In the model with unlimited access, lobbying always decreased the welfare of at least $n-q+1$ legislators and in some cases were unanimously bad for legislator welfare. In equilibria involving an intermediary, lobbying must still decrease the welfare of at least $n-q+1$ legislators but can never decrease the welfare of all legislators. Instead, P's influence increases the welfare of the intermediary along with any other allies in the legislature.

Corollary 2. *In an influential equilibrium with an intermediary the* ex ante *expected utility increases for P's allies and decreases for all other legislators relative to the outcome with no information transmission*

It is possible to observe lobbying without intermediaries even when access is limited. If the cost of lobbying is small enough that the interest group can buy access to all or most legislators, lobbying with no intermediaries is feasible. However, as long as some ally is available to serve as an intermediary, the interest group should not pay the extra costs associated with lobbying a large number of legislators. Lobbying without intermediaries should therefore only be observed when lobbying is costless or when the interest group has no allies in the legislature. Since both of these circumstances are rare in practice, the lion's share of informational lobbying should occur through intermediaries. That is, most informational lobbying should occur through allies.

Corollary 2 also suggests how the model would extend to more realistic assumptions about access costs. For simplicity, the cost c of access is exogenous and does not vary across legislators. Allowing legislators to choose access costs would strengthen the result that interest groups lobby allies since allies would be willing to grant access for zero or even negative costs, while opponents would impose positive costs to compensate for expected policy losses. If c were instead allowed to vary exogenously across legislators, the results would be similar except that costs would induce strict preferences over allies to lobby – in the current model, the interest group is indifferent between allies.

3.3 Extensions and Generalizations

The most restrictive assumptions of the model are the existence of only one lobbyist and the highly specialized informational environment in which payoffs are binary and independently distributed. In the Supplemental Appendix extend the model to include competitive lobbying by two opposing interest groups and show that a group may remain influential in spite of competition and that lobbying allies remains optimal. However, the presence of competition shrinks the region of the parameter space in which a given group can influence the legislature since an opponent can counter the group's lobbying strategy with an opposing signaling strategy (similar to "jamming" equilibria in Minozzi (2011)).

The main results also hold up easily to relaxation of the informational assumptions. The characterizations of existence in Lemma 2 and the welfare implications of influence hold for any state space, distribution, or utility function for the legislators. Lemma 3 and Proposition 2 make use of the independence assumption, but the main effect of the assumption is to simplify the conditions needed for existence of an equilibrium with intermediaries. The assumption of costly access is all that is needed to guarantee that equilibria with intermediaries are optimal when they exist. Finally, Proposition 1 relies on the binary structure of the payoffs, but this is mainly for interpretation: Lemma 2 is all that is required to establish existence.

4 Discussion and Conclusions

The model produces several novel implications of relevance to scholars of interest groups and the policy process. First, lobbying activity should most often be directed at allies. The fact that interest groups tend to focus their attention on allies is generally seen as evidence in contradiction of informational theories of lobbying (Hall and Deardorff, 2006). However informational lobbying is consistent with the data once we consider environments with multiple legislators capable of communicating with each other. In this environment, interest groups should use allies as intermediaries to persuade opponents.

The results share some features with the old models of informational lobbying but in other ways resemble the legislative subsidy theory of Hall and Deardorff. Though lobbying activity is directed at allies, these allies are not the ultimate targets of the interest groups' persuasive efforts. In fact, the information provided by the interest groups tends to be uninformative to the ally legislators about whether or not they should support the proposal. Thus, informational lobbying supports allies' efforts to persuade opponents, occasionally against the best interests of the allies.

Another novel insight of the model is that informational lobbying can be bad for legislators. The informational view of lobbying is often taken to mean that lobbyists provide a service to legislators that increases the quality of representation (Austen-Smith and Wright, 1992). My model suggests that the optimistic conclusions in the literature about interest group influence are tied to the one-legislator models on which most of the literature is based. When the models are adjusted to account for multiple legislators interacting in a collective choice environment, the welfare gains from informational lobbying can be reversed. In fact, as I demonstrate, if interest groups' preferences are independent of the state of the world then the effect of interest group influence on legislator welfare tends to be negative.[20]

Finally, the model suggests than persuading a voting body is a substantively different problem than persuading a single policymaker. The need for voting increases opportunities for persuasion because information can affect which coalitions form to pass or block a proposal. This insight is also critical to other work in political economy. For instance, this insight is found in Bennedsen and Feldmann (2002), one of the few studies other than this one to model informational lobbying with more than one legislator. In their model, an interest group may search for information in each district about demand for a public good, which varies from district to district. Communication is more limited in their model than in this study: for each district, the interest group can either reveal that district's true payoff or reveal nothing. Thus, partial information revelation is not allowed, which focuses their analysis more on information acquisition than on communication. They find that interest groups have a greater incentive to search for information in legislative settings than in those with a single policymaker. The mechanism supporting this conclusion differs from the one in my model since their result relies in part on the way information affects which policies are proposed. Instead, I fix the policy proposal and focus on how lobbying might affect votes.[21] Furthermore, this study differs from Bennedsen and Feldmann by treating legislators asymmetrically to focus on the question of whom interest groups should lobby.

Alonzo and Câmara (2015) also demonstrate how persuasion may work differently in collective choice institutions. Their model is one in which the sender does not have any private information but is able to control the content of a public signal. Despite this difference, the mechanisms for persuasion in their paper are similar to those in Model 1 of this study, and my Lemma 2 follows

[20]This result relies on the assumption that interest group preferences are state-independent. If state-dependent preferences are assumed, welfare effects of lobbying may be positive or negative.

[21]The results from Model 2 are suggestive about how lobbying may work in a model like mine but with endogenous proposals. In such circumstances, the case for lobbying allies would be made even stronger, since the interest group must persuade some legislator to propose a favorable policy in addition to passing on information. A full explication of such a model is left for future work.

from their results. Schnakenberg (2015) also considers a cheap talk setting with voters and shows, as in this paper, that information transmission can negatively affect welfare in voting environments. This paper complements both of these models by including costly access, allowing communication by voters, and considering differences between allies and opponents.

This study complements existing formal models in many ways. The model complements accounts of lobbying as a legislative subsidy by explaining how lobbying of allies may occur in preference-based informational lobbying. Though we would observe interest groups lobbying allies in my model, their intent is to influence opponents. The model also complements vote-buying models by mirroring their focus on collective choice institutions in an informational environment. Furthermore, in comparison to vote-buying and legislative subsidy models, this study better accounts for the prevalence of informational tactics reported in qualitative and survey research on lobbying activities.

Beyond the application to interest groups, the tractable model presented here demonstrates the potential of incorporating collective choice processes into our theories of communication and information transmission in political institutions. Since asymmetric information models have been central to nearly three decades of work on legislative and electoral institutions, the potential applications of the model are voluminous.

References

Ainsworth, Scott H. 1997. "The Role of Legislators in the Determination of Interest Group Influence." *Legislative Studies Quarterly* 22(4):pp. 517–533.

Alonzo, Ricardo and Odilon Câmara. 2015. "Persuading Voters." *Working Paper* .
URL: *http://www-bcf.usc.edu/ ocamara/PersuadingVoters.pdf*

Austen-Smith, David. 1995. "Campaign Contributions and Access." *American Political Science Review* 89:566–581.

Austen-Smith, David. 1998. "Allocating Access for Information and Contributions." *Journal of Law, Economics, & Organization* 14(2):277–303.

Austen-Smith, David and John R. Wright. 1994. "Counteractive Lobbying." *American Journal of Political Science* 38(1):pp. 25–44.

Austen-Smith, David and JohnR. Wright. 1992. "Competitive lobbying for a legislator's vote." *Social Choice and Welfare* 9(3):229–257.

Bauer, Ramond A., Ithiel De Sola Pool and Lewis Anthony Dexter. 1972. *The Governmental Process: Political Interests and Public Opinion*. Aldine Atherton, Inc. Chicago.

Baumgartner, Frank R. and Beth L. Leech. 2001. "Interest Niches and Policy Bandwagons: Patterns of Interest Group Involvement in National Politics." *The Journal of Politics* 63(4):pp. 1191–1213.

Baumgartner, Frank R., Jeffrey M. Berry, Marie Hojnacki, Beth L. Leech and David C. Kimball. 2009. *Lobbying and Policy Change: Who Wins, Who Loses, and Why*. University of Chicago Press.

Bennedsen, Morten and Sven E. Feldmann. 2002. "Lobbying Legislatures." *Journal of Political Economy* 110(4):pp. 919–946.

Browne, William P. 1990. "Organized Interests and Their Issue Niches: A Search for Pluralism in a Policy Domain." *The Journal of Politics* 52(2):pp. 477–509.

Caillaud, Bernard and Jean Tirole. 2007. "Consensus Building: How to Persuade a Group." *The American Economic Review* 97(5):pp. 1877–1900.

Cotton, Christopher. 2009. "Should we tax or cap political contributions? A lobbying model with policy favors and access." *Journal of Public Economics* 93(7):831–842.

Cotton, Christopher. 2012. "Pay-to-play politics: Informational lobbying and contribution limits when money buys access." *Journal of Public Economics* 96(34):369 – 386.

Cotton, Christopher. 2015. "Competing for Attention." *Journal of Public Economic Theory (Forthcoming)* .
URL: *https://ideas.repec.org/p/pra/mprapa/65715.html*

Crawford, Vincent P and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica* 50(6):1431–51.

Denzau, Arthur T. and Michael C. Munger. 1986. "Legislators and Interest Groups: How Unorganized Interests get Represented." *The American Political Science Review* 80(1):pp. 89–106.

Diermeier, Daniel and Roger B. Myerson. 1999. "Bicameralism and Its Consequences for the Internal Organization of Legislatures." *American Economic Review* 89(5):1182–1196.

Drutman, Lee. 2015. *The Business of America is Lobbying: How Corporations Became Politicized and Politics Became More Corporate*. Studies in Postwar American Political Development Oxford University Press.

Drutman, Lee and Daniel J. Hopkins. 2013. "The Inside View: Using the Enron E-mail Archive to Understand Corporate Political Attention." *Legislative Studies Quarterly* 38(1):5–30.
URL: *http://dx.doi.org/10.1111/lsq.12001*

Gray, Virginia and David Lowery. 1997. "Life in a Niche: Mortality Anxiety among Organized Interests in the American States." *Political Research Quarterly* 50(1):pp. 25–47.

Groseclose, Tim and Jr. Snyder, James M. 1996. "Buying Supermajorities." *The American Political Science Review* 90(2):pp. 303–315.

Grossman, Gene M and Elhanan Helpman. 1994. "Protection for Sale." *American Economic Review* 84(4):833–50.

Grossman, G.M. and E. Helpman. 2001. *Special Interest Politics*. Cambridge, MA: MIT Press.

Hall, Richard L. and Alan V. Deardorff. 2006. "Lobbying as Legislative Subsidy." *The American Political Science Review* 100(1):pp. 69–84.

Hansen, John Mark. 1991. *Gaining access : Congress and the farm lobby, 1919-1981 / John Mark Hansen*. University of Chicago Press Chicago.
URL: *http://www.loc.gov/catdir/enhancements/fy0608/91012993-t.html*

Hojnacki, Marie and David C Kimball. 1998. "Organized interests and the decision of whom to lobby in Congress." *American Political Science Review* 92(04):775–790.

Hojnacki, Marie and David C Kimball. 1999. "The who and how of organizations' lobbying strategies in committee." *The Journal of Politics* 61(04):999–1024.

Kamenica, Emir and Matthew Gentzkow. 2011. "Bayesian Persuasion." *American Economic Review* 101(6):2590–2615.

Kollman, Ken. 1997. "Inviting friends to lobby: Interest groups, ideological bias, and congressional committees." *American Journal of Political Science* pp. 519–544.

Kroszner, Randall S and Thomas Stratmann. 2005. "Corporate Campaign Contributions, Repeat Giving, and the Rewards to Legislator Reputation*." *Journal of Law and Economics* 48(1):41–71.

Mahoney, Christine and Frank R Baumgartner. 2015. "Partners in Advocacy: Lobbyists and Government Officials in Washington." *Journal of Politics* .

McCarty, Nolan and Lawrence S. Rothenberg. 1996. "Commitment and the Campaign Contribution Contract." *American Journal of Political Science* 40(3):872–904.

McGinnis, John O and Michael B Rappaport. 1998. "Supermajority rules as a constitutional solution." *Wm. & Mary L. Rev.* 40:365.

Milbrath, L.W. 1963. *The Washington Lobbyists*. Rand McNally.

Minozzi, William. 2011. "A Jamming Theory of Politics." *The Journal of Politics* 73:301–315.

Potters, Jan and Frans Van Winden. 1990. "Modelling political pressure as transmission of information." *European Journal of Political Economy* 6(1):61 – 88.

Potters, Jan and Frans Van Winden. 1992. "Lobbying and asymmetric information." *Public Choice* 74(3):269–292.

Schlozman, K.L. and J.T. Tierney. 1986. *Organized interests and American democracy*. Harper & Row.

Schnakenberg, Keith E. 2015. "Expert advice to a voting body." *Journal of Economic Theory* 160, Part A:102 – 113.
URL: *http://www.sciencedirect.com/science/article/pii/S0022053115001647*

Truman, David B. 1953. *The Governmental Process: Political Interests and Public Opinion*. Alfred A. Knopf, New York.

Wright, John R. 1996. *Interest Groups and Congress: Lobbying, Contributions, and Influence*. Allyn and Bacon.

Appendix: Proofs of Results

Lemma 1 *In any influential equilibrium, the proposal passes with probability one.*

Proof. Assume there is some strategy profile (σ_P, v) such that $x^*(m) = 0$ and $x^*(m') = 1$ for some m and m' in the support of σ_P. Since messages are cheap talk and P's preferences are independent of ω, holding v constant P strictly prefers to deviate from σ_P to a strategy in which m' is sent with probability one following any $\omega \in \Omega$. This shows that such a (σ_P, v) is never an equilibrium, which implies that all influential equilibria lead the proposal to pass with probability one. ■

Lemma 2 *There is an influential equilibrium in Model 1 if and only if $f(\omega, \mathbf{p}) \notin W_q$ and $f(\omega, \mathbf{p}) \in co(W_q)$.*

Proof. This result follows indirectly from various previously proven results (e.g. Lemma 2 of Alonzo and Câmara (2015), Proposition 1 of Kamenica and Gentzkow (2011)) but the proof is reproduced here for the sake of completeness.

First, I will prove that $f(\omega, \mathbf{p}) \in W_q$ and $f(\omega, \mathbf{p}) \in \mathrm{co}(W_q)$ implies the existence of an influential equilibrium. By Caratheodory's theorem, $f(\omega, \mathbf{p}) \in \mathrm{co}(W_q)$ implies that $f(\omega, \mathbf{p})$ is in the convex hull of some finite subset of W_q consisting of $n+1$ or fewer points. Let $\{g^1, \ldots, g^K\} \subset W_q$ denote a set of posterior distributions such that, for some $(\tau^1, \ldots, \tau^K) \in \mathbb{R}_+^K$ such that $\sum_{k=1}^K \tau^k = 1$, we have

$$f(\mathbf{p}, \omega) = \sum_{k=1}^{K} \tau^k g^k(\omega) \tag{3}$$

for all $\omega \in \{0,1\}^n$.

We will define σ_P^* with support on $(m^1, \ldots, m^K)$ as follows:

$$\sigma_P^*(m^k|\omega) = \frac{g^k(\omega)\tau^k}{f(\omega, \mathbf{p})}. \tag{4}$$

The posterior probability of a state ω following a signal m^k is then

$$g(\omega|m^k) = \frac{f(\omega,\mathbf{p})\frac{g^k(\omega)\tau^k}{f(\omega,\mathbf{p})}}{\sum_{\omega'\in\Omega} f(\omega',\mathbf{p})\frac{g^k(\omega')\tau^k}{f(\omega',\mathbf{p})}} = \frac{g^k(\omega)\tau^k}{\sum_{\omega'\in\Omega} g^k(\omega')\tau^k} \tag{5}$$

$$= \frac{g^k(\omega)\tau^k}{\tau^k} = g^k(\omega) \tag{6}$$

Thus, the strategy σ_P^* induces the set of posteriors $\{g^1,\ldots,g^K\} \subset W_q$. By definition of W_q, this implies the the proposal passes with probability one, so this is an equilibrium to the game.

It remains to be shown that the existence of an influential equilibrium implies $f(\omega,\mathbf{p}) \in \mathrm{co}(W_q)$. By way of contraposition, suppose that $f(\omega,\mathbf{p}) \notin \mathrm{co}(W_q)$. By the law of total probability, we have

$$f(\omega,\mathbf{p}) = \sum_{m\in\mathrm{supp}\sigma_P} \Pr[m_P = m] g(\omega|m). \tag{7}$$

Here, $\Pr[m_P = m]$ denotes the total probability (over all states) of $m_P = m$ under the strategy σ_P. That is, $\Pr[m_P = m] = \sum_\Omega f(\omega,\mathbf{p})\sigma(m|\omega)$. By the properties of probability, we have $\sum_{m\in\mathrm{supp}\sigma_P}\Pr[m_P = m] = 1$. However, since $f(\omega,\mathbf{p}) \notin \mathrm{co}(W_q)$ there is no way to express $f(\omega,\mathbf{p})$ as a convex combination of distributions in W_q. Thus, at least one posterior distribution is not an element of W_q. In other words, there is no σ_P that always guarantees passage of the proposal. By Lemma 1, this implies that there is no influential equilibrium. ■

Proposition 1 *In Model 1:*

1. *Let $\hat{\mathbf{p}} \geq \mathbf{p}$ and assume $f(\omega,\hat{p}) \notin W_q$. If there is an influential equilibrium when prior beliefs are $f(\omega,\mathbf{p})$ then there is also an influential equilibrium when prior beliefs are $f(\omega,\hat{\mathbf{p}})$, holding everything else equal.*

2. *There exists $p^* \in \left(0,\frac{L}{H+L}\right)$ such that if $p_i \geq p^*$ for all $i \in N$ then there exists an equilibrium in which the proposal always passes.*

Proof. To prove part (1), I must show that $f(\omega,\mathbf{p}) \in \mathrm{co}(W_q)$ and $\hat{\mathbf{p}} \geq \mathbf{p}$ implies $f(\omega,\hat{\mathbf{p}}) \in \mathrm{co}(W_q)$. Thus, let $f(\omega,\mathbf{p}) \in \mathrm{co}(W_q)$. By Caratheodory's theorem, $f(\omega,\mathbf{p})$ is in the convex hull of some finite subset of W_q. Let $R = \{g_1, g_2, \ldots, g_{|R|}\} \subset W_q$ be one such finite set. Then there exists $\tau = (\tau_1, \ldots, \tau_{|R|})$ be such that $\tau \geq \mathbf{0}$, $\sum_{j=1}^{|R|} \tau_j = 1$, and $\sum_{j=1}^{|R|} \tau_j g_j(\omega) = f(\mathbf{p}, \omega)$ for all $\omega \in \{0,1\}^n$.

It suffices to show that the result holds for distributions created by increasing only one of the probabilities in $\mathbf{p}$ while holding the rest constant, since any $\hat{\mathbf{p}} > \mathbf{p}$ can be generated by a series of such changes. Without loss of generality we will change only the probability associated with voter 1. Let $\mathbf{p}' = (p_1', p_2, \ldots, p_n)$ be a vector in which voter 1's probability is increased to $p_1' > p_1$ and the other probabilities remain the same.

Let $\mathbf{p}'' = (1, p_2, \ldots, p_n)$ be a vector in which voter one's probability is increased to 1 and the other probabilities remain the same. Let $\psi = (1, 0, 0, \ldots, 0)$. Thus, if $\omega_1 = 1$ then $\omega - \psi$ is a state in which $\omega_1 = 0$ and all other elements are unchanged. Define $R' = \{g_1', g_2', \ldots, g_{|R|}'\}$ such that

$$g_j'(\omega) = \begin{cases} g_j(\omega) + g_j(\omega - \psi) & \text{if } \omega_1 = 1 \\ 0 & \text{if } \omega_1 = 0 \end{cases} \tag{8}$$

for all $j \in \{1, \ldots, |R|\}$. For all $i \neq 1$ and $j \in \{1, \ldots, |R|\}$, we have

$$\begin{aligned}
\mathbb{E}_{g_j'}[\omega_i] &= \sum_{\omega : \omega_i = 1} g_j'(\omega) && (9) \\
&= \sum_{\substack{\omega' : \omega_i' = 1 \\ \wedge \omega_1' = 1}} \left[g_j(\omega') + g_j(\omega' - \psi) \right] && (10) \\
&= \sum_{\substack{\omega' : \omega_i' = 1 \\ \wedge \omega_1' = 1}} g_j(\omega') + \sum_{\substack{\omega' : \omega_i' = 1 \\ \wedge \omega_1' = 0}} g_j(\hat{\omega}) && (11) \\
&= \sum_{\omega : \omega_i = 1} g_j(\omega) && (12) \\
&= \mathbb{E}_{g_j}[\omega_i]. && (13)
\end{aligned}$$

Thus, $R \subset W_q$ implies $R' \subset W_q$.

Since

$$\sum_{j=1}^{|R|} \tau_j g_j(\omega) = f(\omega, \mathbf{p}) = \prod_{i \in N} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i}, \tag{14}$$

for each ω such that $\omega_1 = 1$ we have

$$\sum_{j=1}^{|R|} \tau_j g'_j(\omega) = \sum_{j=1}^{|R|} \tau_j (g_j(\omega) + g_j(\omega - \psi)) \tag{15}$$

$$= f(\omega, \mathbf{p}) + f(\omega - \psi, \mathbf{p}) \tag{16}$$

$$= \prod_{i \in N} p_i^{\omega_i} (1 - p_i)^{1-\omega_i} + \prod_{i \in N} p_i^{\omega_i - \psi} (1 - p_i)^{1-\omega_i-\psi} \tag{17}$$

$$= p_1 \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1-\omega_i} + (1 - p_1) \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1-\omega_i} \tag{18}$$

$$= \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1-\omega_i} \tag{19}$$

$$= f(\omega, \mathbf{p}'') \tag{20}$$

which shows that $f(\omega, \mathbf{p}'') \in \text{co}(R') \subset \text{co}(W_q)$. Furthermore, since $\text{co}(W_q)$ is a convex set and $\mathbf{p}'$ is a convex combination of $\mathbf{p}$ and $\mathbf{p}''$, we have $f(\omega, \mathbf{p}') \in \text{co}(W_q)$. This implies that $f(\omega, \hat{\mathbf{p}}) \in \text{co}(W_q)$ for all $\hat{\mathbf{p}} > \mathbf{p}$, which completes the proof for part (1).

To prove part (2) I will first assume that $p_i = p_j = \overline{p}$ for all $i, j \in N$ and find a lower bound on $\overline{p}$ such that there exists an equilibrium in which the proposal passes with probability one. Part (1) of this proposition then implies that the proposal always passes $p_i > \overline{p}$ for all $i \in N$, even when prior probabilities are not assumed to be equal.

For any $\omega \in \Omega$, let $B(\omega) = \{i \in N : \omega_i = 1\}$. Consider the following strategy for P:

$$\sigma_p^*(m|\omega) = \begin{cases} \binom{n-|B(\omega)|}{q-|B(\omega)|}^{-1} & \text{if } |B(m)| = q \text{ and } B(\omega) \subset B(m) \\ \binom{|B(\omega)|}{q}^{-1} & \text{if } |B(m)| = q \text{ and} B(\omega) \supset B(m) \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

In words, Equation 21 means that P always recommends approval to a minimal winning coalition, and does so in the following way:

- When the number of true beneficiaries is weakly smaller than q, P randomizes uniformly among all minimal winning coalitions containing the set of true beneficiaries.

- When the number of true beneficiares is strictly larger than q, P randomizes uniformly among all minimal winning coalitions that are subsets of the set of true beneficiaries.

The prior distribution over the number of true beneficiaries is the binomial distribution:

$$\Pr[|B(\omega)| = k] \equiv b(k, \overline{p}) = \binom{n}{k} \overline{p}^k (1-\overline{p})^{n-k}. \tag{22}$$

Note that σ_P^* provides no information about the size of the set of true beneficiaries. The posterior distribution following each m in the support of σ_P^* is

$$g(\omega|m) = \begin{cases} b(|B(\omega)|, \overline{p}) \binom{q}{|B(\omega)|}^{-1} & \text{if } B(\omega) \subseteq B(m) \\ b(|B(\omega)|, \overline{p}) \binom{|B(\omega)|}{q}^{-1} & \text{if } B(\omega) \supset B(m) \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Therefore, the probability that $\omega_i = 1$ given $i \in B(m)$ is

$$\Pr[\omega_i = 1 | i \in B(m)] = \sum_{k=0}^{q} b(k, \overline{p}) \binom{q-1}{k-1} \binom{q}{k}^{-1} + \sum_{k=q+1}^{n} b(k, \overline{p}) \tag{24}$$

$$= \sum_{k=0}^{q} b(k, \overline{p}) \frac{k}{q} + \sum_{k=q+1}^{n} b(k, \overline{p}) \tag{25}$$

$$= \sum_{k=0}^{n} b(k, \overline{p}) \min\{\frac{k}{q}, 1\}. \tag{26}$$

Let $\varphi(\overline{p}) = \sum_{k=0}^{n} b(k, \overline{p}) \min\{\frac{k}{q}, 1\}$. Let p^* be the probability solving

$$\varphi(p^*) = \frac{L}{H+L}.$$

Such a p^* exists and is unique since φ is continuous and monotone increasing with $\varphi(0) = 0$ and $\varphi(1) = 1$. Thus, when $\overline{p} \geq p^*$, we have $\pi_i(M) > \frac{L}{H+L}$ for all $i \in B(m)$ following every message m in the support of this strategy. Since each m targets q legislators, this implies that the proposal always passes. Thus, there is a persuasive equilibrium when $p_i = p_j = \overline{p} \geq p^*$ for all $i, j \in N$. Furthermore, part (1) of the proposition implies that this holds for any distribution such that $p_i \geq p^*$ for all $i \in N$.

■

Lemma 3 *Assume that $p_i \geq L/(H+L)$ for some i. Then any influential equilibrium is outcome-equivalent to one in which $\pi_i(m) = p_i$ for all $m \in \Omega$.*

Proof. Assume there exists an influential equilibrium and that $p_1 \geq L/(H+L)$. Then by Lemma 2 there are a set of posteriors $R = (g_1, \ldots, g_{|R|}) \subset W_q$ and $(\tau_1, \ldots, \tau_{|R|})$ in the $(|R|-1)$-dimensional simplex such that

$$f(\omega, \mathbf{p}) = \sum_{j=1}^{|R|} \tau_j g_j(\omega). \tag{27}$$

Let $\psi = (1, 0, 0, \ldots, 0)$. So, if $\omega_1 = 1$ then $\omega - \psi$ is a state with $\omega_1 = 0$ and all other elements unchanged and if $\omega_1 = 0$ then $\omega + \psi$ is a state with $\omega_1 = 1$ and all other elements unchanged. For each $j \in \{1, \ldots, |R|\}$, we will construct a new posterior distribution g'_j as follows

$$g'_j(\omega) = \begin{cases} (g_j(\omega) + g_j(\omega - \psi))p_1 & \text{if } \omega_1 = 1 \\ (g_j(\omega) + g_j(\omega + \psi))(1 - p_1) & \text{if } \omega_1 = 0. \end{cases} \tag{28}$$

We need to verify that

(a) legislator 1's probability of benefiting in each posterior is equal to p_1,

(b) $\{g'_1, \ldots, g'_{|R|}\} \subset W_q$, and

(c) $f(\omega, \mathbf{p}) \in \text{co}(\{g'_1, \ldots, g'_{|R|}\})$.

Part (a) is easily verified – since $\sum_{\omega \in \Omega : \omega_1 = 1}(g_j(\omega) + g_j(\omega - \psi)) = 1$, we have $\sum_{\omega \in \Omega : \omega_1 = 1}(g_j(\omega) + g_j(\omega - \psi))p_1 = p_1$. To prove part (b) note that, for all $i \neq 1$, the probability of benefiting according to the posterior g'_j is:

$$\sum_{\omega \in \Omega: \omega_i = 1} g'_j(\omega) = p_1 \sum_{\substack{\omega' \in \Omega: \omega'_i = 1 \\ \wedge \omega'_1 = 1}} [g_j(\omega') + g_j(\omega' - \psi)] + (1 - p_1) \sum_{\substack{\omega'' \in \Omega: \omega''_i = 1 \\ \wedge \omega''_1 = 0}} [g_j(\omega'') + g_j(\omega'' + \psi)] \tag{29}$$

$$= p_1 \sum_{\substack{\omega' \in \Omega: \omega'_i = 1 \\ \wedge \omega'_1 = 1}} g_j(\omega') + p_1 \sum_{\substack{\omega'' \in \Omega: \omega''_i = 1 \\ \wedge \omega''_1 = 0}} g_j(\omega'') + (1 - p_1) \sum_{\substack{\omega'' \in \Omega: \omega''_i = 1 \\ \wedge \omega''_1 = 0}} g_j(\omega'') + (1 - p_1) \sum_{\substack{\omega' \in \Omega: \omega'_i = 1 \\ \wedge \omega'_1 = 1}} g_j(\omega') \tag{30}$$

$$= [p_1 + (1 - p_1)] \sum_{\substack{\omega' \in \Omega: \omega'_i = 1 \\ \wedge \omega'_1 = 1}} g(\omega') + [p_1 + (1 - p_1)] \sum_{\substack{\omega'' \in \Omega: \omega''_i = 1 \\ \wedge \omega''_1 = 0}} g(\omega'') \tag{31}$$

$$= \sum_{\omega \in \Omega: \omega_i = 1} g_j(\omega). \tag{32}$$

Thus, since legislator 1 is always incentivized to vote in favor and the expected benefits to all other legislators in each posterior are the same as under the distributions in R, we have $\{g'_1, \ldots, g'_{|R|}\} \subset W_q$.

Finally, we must verify part (c), that the prior distribution is in the convex hull of $\{g'_1, \ldots, g'_{|R|}\}$. For all $\omega \in \Omega$, we have for each $\omega \in \Omega$ such that $\omega_1 = 1$:

$$\sum_{j=1}^{|R|} \tau_j g'_j(\omega) = \sum_{j=1}^{|R|} \tau_j [g_j(\omega) + g_j(\omega - \psi)] p_1 \tag{33}$$

$$= p_1 \left[\sum_{j=1}^{|R|} \tau_j g_j(\omega) + \sum_{j=1}^{|R|} \tau_j g_j(\omega - \psi) \right] \tag{34}$$

$$= p_1 \left[f(\omega, \mathbf{p}) - f(\omega - \psi, \mathbf{p}) \right] \tag{35}$$

$$= p_1 p_1 \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} + (1 - p_1) p_1 \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} \tag{36}$$

$$= p_1 \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} \tag{37}$$

$$= f(\omega, \mathbf{p}). \tag{38}$$

Where line 35 follows from that fact that $f(\omega, \mathbf{p}) = \sum_{j=1}^{|R|} \tau_j g_j(\omega)$., for all $\omega \in \Omega$. Similarly, for each $\omega \in \Omega$ such that $\omega_1 = 0$:

$$\sum_{j=1}^{|R|} \tau_j g'_j(\omega) = \sum_{j=1}^{|R|} \tau_j [g_j(\omega) + g_j(\omega + \psi)](1 - p_1) \tag{39}$$

$$= (1 - p_1) \left[\sum_{j=1}^{|R|} \tau_j g_j(\omega) + \sum_{j=1}^{|R|} \tau_j g_j(\omega + \psi) \right] \tag{40}$$

$$= (1 - p_1) [f(\omega, \mathbf{p}) + f(\omega + \psi, \mathbf{p})] \tag{41}$$

$$= (1 - p_1)(1 - p_1) \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} + (1 - p_1) p_1 \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} \tag{42}$$

$$= (1 - p_1) \prod_{i \in N \setminus \{1\}} p_i^{\omega_i} (1 - p_i)^{1 - \omega_i} \tag{43}$$

$$= f(\omega, \mathbf{p}). \tag{44}$$

Thus, we have $f(\omega, \mathbf{p}) \in \mathrm{co}(\{g'_1, \ldots, g'_{|R|}\})$. ■

Proposition 2 *Assume that $c > 0$ and that P has at least one ally (that is, $p_i > L/(H+L)$ for some i in N). Then P's optimal influential equilibrium has the following properties:*

1. *P lobbies exactly one legislator and that legislator is an ally, and*

2. *The lobbied legislator always conveys P's message to the legislature and always vote in favor of passage*

Proof. For some i, assume that $p_i > L/(H+L)$. Let $A_P = \{i\}$ and assume that σ_P^* is such that $\pi_i(m) = p_i$ following any message and $\{g(\omega|m)\}_{m \in \mathrm{supp}(\sigma_P^*)} \subset W_q$. By Lemma 3, such a messaging strategy exists whenever there is an influential equilibrium. Since i strictly prefers passing the proposal at each information set, i should choose to convey each message to the legislature. Thus, an influential equilibrium can be supported by only lobbying only i, which is strictly preferable to lobbying more than one legislator. Since P is assumed to choose the payoff-maximizing strategy, this is true of any influential equilibrium. Furthermore, this arrangement cannot be an equilibrium if $A_P = \{j\}$ where j is such that $p_j < L/(H+L)$, since j will have an incentive to conceal P's message at some information sets for any distribution of posteriors with $f(\omega, \mathbf{p})$ in their convex hull.

To complete the equilibrium characterization, I will conclude by describing a full strategy profile with off-path beliefs. Let v_j^f denote legislator j's optimal vote under the prior. That is $v_j^f ='' Yes''$ if $p_i \geq L/(H+L)$ and "No" otherwise. Without loss of generality, we will label the messages in the support of σ_P^* such that $\pi_j(m) \geq L/(H+L)$ if and only if j'th element of m is equal to 1. The following is a full strategy profile for P's optimal equilibrium:

- $A_P = \{i\}$, $\sigma_P = \sigma_P^*$ (from above)
- $M_i(A) = \text{supp}(\sigma_P^*)$ for all $A \subset N$
- $v_j(0) = v_j^f$ and
$$v_j(1) = \begin{cases} \text{Yes} & \text{if } m_j = 1 \\ \text{No} & \text{if } m_j = 0 \end{cases}$$
for all $j \in N \backslash \{i\}$
- $$v_i(m) = \begin{cases} \text{Yes} & \text{if } m_i = 1 \\ \text{No} & \text{if } m_i = 0 \end{cases}.$$
- All off-path beliefs are equal to $f(\omega, \mathbf{p})$.

This specification of off-path beliefs and actions is only one example of an equilibrium construction that supports σ_P^* and M_i. Other payoff-equivalent specifications are possible. ■