Informational Lobbying and Legislative Voting: Supplemental Information

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The main text concerns situations in which only one lobbyist attempts to influence the legislature. In many cases, however, there may be lobbyists on both sides of an issue sending counteractant messages to legislators. I now turn to the case of two interest groups. The goal of the analysis is to show that influence is still possible in the presence of an opposing interest group and that the main empirical predictions of the model still hold true.

In the competitive model, the set of players includes an interest group O representing opponents of the proposed policy in addition to the original proponent interest group P and the set N of legislators. O's policy payoffs are the opposite of P's but the per-legislator lobbying costs are the same between P and O. The sequence of play is such that O observes the lobbying and messaging choices of P before deciding on its own move. The decision to let O move second is based on O's position in the game: Given that O is in the position of supporting the status quo policy and the one that would prevail if neither interest group engaged in lobbying, it is less likely in practice that O would move first since only P *must* lobby to get its preferred outcome.

The sequence of play is otherwise similar to Model 2. First, the interest groups choose which legislators to which to invest in access. P makes this decision first. O observes P's decision and then makes its decision. Let A_P and A_O denote the sets of legislators accessed by P and O, respectively. Next, both interest groups learn the state of the world and the communication stage begins: Initially P sends a cheap talk message m_P to members of A_P and members of A_P decide whether to make that message public. Following these choices, O observes all messages and chooses a message m_O to send to members of A_O. The legislators in A_O also decide whether to make O's message public. Finally, legislators vote on the final policy as in the original model.

The utility of each legislator is

$$u_i(x,\omega) = \begin{cases} H & \text{if } x = 1 \text{ and } \omega_i = 1 \\ -L & \text{if } x = 1 \text{ and } \omega_i = 0 \\ 0 & \text{if } x = 0. \end{cases} \tag{1}$$

as in the previous models. P's utility function is $u_P(x,A_P) = x - c|A_P|$ and O's utility function is $u_O(x,A_O) = 1 - x - c|A_O|$ where $c \in [0,1)$.

I start by proving an analog of Lemma 1 for the competitive lobbying case. Since we are focused on influence by P, an influential equilibrium is still one in which the proposal would not have passed prior to any lobbying but passes with some positive probability in equilibrium. As in the single interest group case, it follows that the proposal must pass with probability one in any influential equilibrium.

Lemma 4. *In any influential equilibrium to the competitive lobbying game, the proposal passes with probability one.*

Proof. Following any particular message from P, the probability that the proposal passes must be zero or one. To see this, suppose there is some strategy profile (σ_P, σ_O, v) and messages m, m' and m'' in Ω such that:

- m is in the support of σ_P and m' and m'' are in the support of σ_O following $m_p = m$,
- when $m_P = m$ and $m_O = m'$, we have $x = 0$, and
- when $m_P = m$ and $m_O = m''$, we have $x = 1$.

Since O's preferences are independent of the state and m_O is cheap talk, holding v constant O strictly prefers to deviate to a profile in which m' is sent with probability one. This shows that the proposal must pass with probability one or zero following a particular realization of m_P. Thus, the same logic as in Lemma 1 implies that the proposal must pass with probability one in any influential equilibrium: if some messages lead to passage of the proposal and other messages lead to failure of the proposal, then P would deviate to a profile that only sends messages leading to passage. Therefore, all messages must lead to the same outcome, implying that the proposal passes with probability one in any influential equilibrium.■

For the main analysis, I will start once again with the case of costless lobbying ($c = 0$) in order to analyze the communication problem in isolation. When $c = 0$, both interest groups are assumed to have access to all legislators and communication by legislators is irrelevant, so I will temporarily ignore those decisions and analyze only the interest groups' messages and the legislators' voting strategies. In this game P is at a disadvantage in two significant ways: the voting rule (especially larger super-majority rules) discourages changing the status quo and O is allowed to observe P's actions before making lobbying or messaging decisions. Nevertheless, there exist parameters for which P can persuade the legislature to pass the proposal and O is unable to block P's influence. Example 3 demonstrates this point using another three-legislator case.

Example 3. Let $n = 3$ and $q = 2$. Assume that $H = 200$, $L = 800$, and $p_1 = p_2 = p_3 = {}^3/_4$. With no lobbying, the proposal would fail unanimously since legislators should only approve the proposal if they expect to benefit with a probability of at least ${}^4/_5$. Let P's messaging strategy be the same as in Example 1: if all legislators or no legislators benefit then P chooses a random minimal winning coalition, if only legislator i benefits then P chooses randomly between the minimal winning coalitions that include i, and if exactly two legislators benefit then P certainly chooses the minimal winning coalition including the two beneficiaries. The calculations for the posterior beliefs following this first stage of communication are given in Table 1. The two targeted legislators now believe that they benefit from the proposal with probability ${}^{117}/_{128}$ and the remaining legislator believes that she benefits with probability ${}^{27}/_{64}$. Thus, if communication stop at this point, the proposal will pass.

However, O is given an opportunity to send messages that may prevent the proposal from passing. The best possible strategy for O would be the one that targets only legislators already

ω	$\sigma_P(\{1,2\}\vert\omega)$	$\sigma_P(\{1,3\}\vert\omega)$	$\sigma_P(\{2,3\}\vert\omega)$	$f(\omega,\mathbf{p})$	$\Pr[\omega\vert\{1,2\}]$	$\Pr[\omega\vert\{1,3\}]$	$\Pr[\omega\vert\{2,3\}]$
$\{0,0,0\}$	$^{1}/_{3}$	$^{1}/_{3}$	$^{1}/_{3}$	$^{1}/_{64}$	$^{1}/_{64}$	$^{1}/_{64}$	$^{1}/_{64}$
$\{0,0,1\}$	0	$^{1}/_{2}$	$^{1}/_{2}$	$^{3}/_{64}$	0	$^{3}/_{128}$	$^{3}/_{128}$
$\{0,1,0\}$	$^{1}/_{2}$	0	$^{1}/_{2}$	$^{3}/_{64}$	$^{3}/_{128}$	0	$^{3}/_{128}$
$\{0,1,1\}$	0	0	1	$^{9}/_{64}$	0	0	$^{27}/_{64}$
$\{1,0,0\}$	$^{1}/_{2}$	$^{1}/_{2}$	0	$^{3}/_{64}$	$^{3}/_{128}$	$^{3}/_{128}$	0
$\{1,0,1\}$	0	1	0	$^{9}/_{64}$	0	$^{27}/_{64}$	0
$\{1,1,0\}$	1	0	0	$^{9}/_{64}$	$^{27}/_{64}$	0	0
$\{1,1,1\}$	$^{1}/_{3}$	$^{1}/_{3}$	$^{1}/_{3}$	$^{27}/_{64}$	$^{27}/_{64}$	$^{27}/_{64}$	$^{27}/_{64}$
					$M=\{1,2\}$	$M=\{1,3\}$	$M=\{2,3\}$
				$\pi_1(M)$:	$\mathbf{^{117}/_{128}}$	$\mathbf{^{117}/_{128}}$	$^{27}/_{64}$
				$\pi_2(M)$:	$\mathbf{^{117}/_{128}}$	$^{27}/_{64}$	$\mathbf{^{117}/_{128}}$
				$\pi_3(M)$:	$^{27}/_{64}$	$\mathbf{^{117}/_{128}}$	$\mathbf{^{117}/_{128}}$

Table 1: Calculations for the first communication stage of Example 3. In the top half of the table: The first column lists each possible state of the world. The second, third and fourth columns show the probabilities of the messages $\{1,2\}$, $\{1,3\}$ and $\{2,3\}$ (respectively) given each state of the world. The fifth column shows the prior probability of each state of the world. The sixth, seventh, and eighth columns show the posterior probability of each state of the world given the messages $\{1,2\}$, $\{1,3\}$, and $\{2,3\}$ (respectively). Since the total probability of each message is $^{1}/_{3}$, each posterior following each message M is $3\cdot(\sigma_p(M|\omega)f(\omega,\mathbf{p})$. The bottom half of the table shows each legislators' posterior probability of benefiting form the policy following each message, calculated for each i by adding up the posterior probabilities of all states for which $\omega_i=1$. Posterior probabilities that lead the legislator to vote in favor of passage are highlighted in **bold**.

targeted by P and then minimizes the posterior probability of benefiting for the targeted legislator. Applying Bayes' rule tells us that the latter goal is achieved by minimizing the probability of targeting a legislator for whom $\omega_i=1$ and maximizing the probability of targeting a legislator for whom $\omega_i=0$. Ideally O would never target legislators for whom $\omega_i=1$ but this cannot be credible since there are states with two or three beneficiaries and in those cases $\omega_i=1$ for all $i\in M_P$. Thus, the best that O can do is only target beneficiaries when there are two or three beneficiaries and to always target non-beneficiaries when there are 0 or 1. Thus, O's best strategy targets one random legislator in M_P when there are 0, 2, or 3 beneficiaries and targets the non-beneficiary in M_P when there is exactly one beneficiary. This means that the probability of benefiting for the legislator targeted by both interest groups is exactly equal to the the prior probability that there are at least two beneficiaries. In this case, that means the posterior probability of benefiting is $^{27}/_{32}$, which is less than the posterior probability when only P lobbies but still greater than the $^{4}/_{5}$ threshold required for the legislator to support the proposal. Thus, O fails to block passage of the proposal. ■

Example 3 demonstrates that P may be able to persuade the legislature to pass the proposal even when the legislators are unanimously opposed to the policy *ex ante* and O has an opportunity to counteract P's attempts at persuasion. In other cases, O can block passage of the proposal when P otherwise would have been influential.[1]

[1] For instance, the same blocking strategy that was unsuccessful in Example 3 would have successfully blocked

In the competitive lobbying game, an influential equilibrium is one in which P can persuade the legislature to pass the proposal and O is unable to prevent passage. A blocking equilibrium, in contrast, is one in which P could have been influential in the absence of O, but O is able to prevent passage. Lemma 5 shows necessary and sufficient conditions for each type of equilibrium. Recall that W_q is the set of beliefs for which the proposal passes and let R_q denote the set of beliefs for which the proposal is rejected. According to Lemma 1, in an influential equilibrium O must be unable to block passage of the proposal following *any* message that P sends. Thus, the necessary and sufficient conditions for influential and blocking equilibria are directly related to those in Lemma 2. In an influential equilibrium, the conditions from Lemma 2 must hold *and* the comparable condition of O must not hold for any posterior distribution. As before, the result boils down to whether the prior distribution is in the convex hull of the desired set of posterior distributions.

Lemma 5. *In the competitive lobbying model with $c = 0$, P is influential if and only if $f(\omega, \mathbf{p}) \in co(W_q \backslash co(R_q))$. P is blocked by O if and only if $f(\omega, \mathbf{p}) \in co(W_q)$ but $f(\omega, \mathbf{p}) \notin co(W_q \backslash co(R_q))$.*

Proof. The result follows directly from Lemma 1 and Lemma 2: By Lemma 1, there is not an influential equilibrium if O is able to block passage following one of the messages. Applying Lemma 2, O can block passage of the proposal following any message that generates a posterior in the convex hull of R_q. Thus, an influential equilibrium occurs if and only if there is a feasible distribution of posteriors in the winset W_q but not in the convex hull of the rejection set R_q, or when $f(\omega, \mathbf{p}) \in \mathrm{co}(W_q \backslash \mathrm{co}(R_q))$. Conversely, a blocking equilibrium occurs when an influential equilibrium would exist in the one-lobbyist game but can be blocked in the two-lobbyist game: that is true if and only if $f(\omega, \mathbf{p}) \in \mathrm{co}(W_q)$ but $f(\omega, \mathbf{p}) \notin \mathrm{co}(W_q \backslash \mathrm{co}(R_q))$. ■

Lemma 5 gives way to a more easily interpretable sufficient condition for a blocking equilibrium which mirrors the explanation of Example 3. If the prior probability that there are at least q true beneficiaries is less than the threshold probability needed for a legislator to support the proposal, then O can always prevent the proposal from passing.

Proposition 3. *If $\Pr[\sum_{i \in N} \omega_i \geq q] < L/(H+L)$ then O can always prevent the proposal from passing.*

Proof. Consider a strategy σ_O such that $|B(m_O)| = n - q + 1$ for all $m_O \in \mathrm{supp}\sigma_O$, and $\sigma_O(\tilde{m}|\omega) = 0$ if $|B(\omega)| < q$ and $B(\tilde{m}) \cap B(\omega) \neq \emptyset$. In other words, when there are fewer than q winners no winner is ever targeted by O. Thus, for each $i \in M_O$,

$$\pi_i(M_P, M_O) = \Pr[|B(\omega)| < q | m_O] \Pr[\omega_i = 1 | m_P, m_O, |B(\omega)| < q] + \Pr[|B(\omega)| \geq q | m_O] \Pr[\omega_i = 1 | m_P, m_O, |B(\omega)| \geq q] \tag{2}$$

$$= 0 + \Pr[|B(\omega)| \geq q | m_O] \Pr[\omega_i = 1 | m_P, m_O, |B(\omega)| \geq q] \tag{3}$$

$$\leq \Pr[|B(\omega)| \geq q | m_O]. \tag{4}$$

Therefore, if $\Pr[\sum_{i \in N} \omega_i \geq q] < L/(H+L)$ then $n - q + 1$ legislators vote against the proposal following any lobbying message from O. ■

When lobbying is costly and both sides have allies in the legislature, the conditions for the existence of influential equilibria do not change considerably relative to the case where $c = 0$. If

passage of the proposal under the parameters from Example 1.

the conditions for existence of influential equilibria from Lemma 5 are met and P has an ally in the legislature then there is an influential equilibrium in which P uses an ally as an intermediary and it follows the same logic as in 2. When no allies are available to P, as in Example 3, the level of lobbying required to influence the policy outcome may be so costly that P prefers to let the proposal fail.

Under the conditions of Proposition 3, there is a blocking equilibrium as in the case where $c = 0$. However, when lobbying is costly, a blocking equilibrium is observationally equivalent to equilibria in which no productive lobbying is possible. In a blocking equilibrium, all lobbying occurs off the equilibrium path: If P chose to lobby, so would O. However, since O's lobbying efforts would surely prevent the proposal from passing, P chooses not to incur the costs associated with lobbying. Furthermore, since the proposal surely fails when P does not lobby, O will also choose not to lobby.

Proposition 4. *Let $c > 0$ and assume that $p_i > L/(H+L)$ and $p_j < L/(H+L)$ for some $i, j \in N$. The following hold for the competitive lobbying game:*

1. *If $f(\omega, \mathbf{p}) \in co(W_q \backslash co(R_q))$ then there is an influential equilibrium in which P lobbies a single legislator, the lobbied legislator is an ally and always conveys P's message to the legislature, and O does not engage in lobbying.*

2. *If $\Pr[\sum_{i \in N} \omega_i \geq q] < L/(H+L)$ then there is a blocking equilibrium. In this equilibrium, neither interest group engages in lobbying on the path of play. Off the equilibrium path, O lobbies a single legislator who is O's ally and always conveys O's message to the legislature.*

Proof. Part (1) follows directly from Lemma 5. The existence of a blocking equilibrium when $\Pr[\sum_{i \in N} \omega_i \geq q] < L/(H+L)$ follows from Proposition 3. Since $c \in (0,1)$, O lobbies one legislator if A_P is non-empty and employs a successful blocking strategy and chooses $A_O = \emptyset$ if $A_P = \emptyset$. Since no lobbying decision by P leads the proposal to pass and $c > 0$, P chooses $A_P = \emptyset$ in equilibrium. ■

The second part of Proposition 4 raises an important point about the impact of lobbying groups. Though neither group chooses to lobby in the blocking equilibrium, the fact that O is organized and able to lobby changes the policy outcome. In that sense, members of an interest group benefit from the capacity of the group to engage in lobbying even when active lobbying is not necessary or desirable.

The proof Proposition 4 did not rely on a full specification of the strategy profile. However, many different strategy profiles may support the equilibrium described in Proposition 4. For instance, the equilibrium in the first part of Proposition 4 can be supported using the strategy profile described in the proof of Proposition 2, setting $A_O = \emptyset$ and all off-path beliefs equal to the prior (this assumes the case where P might be influential, which is to say that the outcome with no lobbying is that the proposal fails). The equilibrium in the second part of Proposition 4 can be supported by setting $A_P = \emptyset$,

$$A_O(A_P) = \begin{cases} \emptyset & \text{if } A_P = \emptyset \\ \{j\} & \text{otherwise.} \end{cases}$$

where $p_j < L/(H+L)$, σ_O equal to a blocking strategy (which we have shown to exist for any σ_P off the path of play under the conditions of this equilibrium), $M_j(A) = \text{supp}(\sigma_O)$, and setting v equal the appropriate Bayes-consistent voting strategies off the equilibrium path. The critical insight for the second type of information does not rely on a particular strategy profile but on the fact that any messaging strategy by P would be blocked by O under these conditions. Note once again that all lobbying is off the path of play. Thus, in an empirical setting, O's influence on policy could not be measured by simply observing the situations in which O actually engaged in lobbying. The fact that O is organized and prepared to lobby may change the outcome even when no lobbying is observed.